5) $\mathcal{A} = \left(\begin{array}{c|c} A & 0 \\ \hline B & E \end{array}\right)$; $\left(\begin{array}{c|c} A & 0 \\ \hline B & E \end{array}\right) \cdot \overbrace{\left(\begin{array}{c|c} X_1 & X_3 \\ \hline X_2 & X_4 \end{array}\right)}^{\mathcal{A}^{-1}} = \left(\begin{array}{c|c} E & 0 \\ \hline 0 & E \end{array}\right)$

$A \overset{!}{}$ regulär | $(A \mid 0) \cdot \left(\begin{array}{c} X_1 \\ \hline X_2 \end{array}\right) = A \cdot X_1 = E \rightarrow X_1 = A^{-1}$

$(A \mid 0) \cdot \left(\begin{array}{c} X_3 \\ \hline X_4 \end{array}\right) = A \cdot X_3 = 0 \rightarrow X_3 = 0 \rightarrow \mathcal{A}^{-1} = \left(\begin{array}{c|c} A^{-1} & 0 \\ \hline X_2 & X_4 \end{array}\right)$

$(B \mid E) \cdot \left(\begin{array}{c} 0 \\ \hline X_4 \end{array}\right) = X_4 = E \rightarrow \mathcal{A}^{-1} = \left(\begin{array}{c|c} A^{-1} & 0 \\ \hline X_2 & E \end{array}\right)$

$(B \mid E) \cdot \left(\begin{array}{c} A^{-1} \\ X_2 \end{array}\right) = BA^{-1} + X_2 = 0 \rightarrow X_2 = -BA^{-1}$

$$A^{-1} = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{4}{3} \\ -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathcal{A}^{-1} = \left(\begin{array}{c|c} A^{-1} & 0 \\ \hline -BA^{-1} & E \end{array}\right) = \left[\begin{array}{ccc|ccc} \frac{2}{3} & -\frac{1}{3} & -\frac{4}{3} & 0 & 0 & 0 \\ -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline -\frac{5}{3} & -\frac{2}{3} & \frac{4}{3} & 1 & 0 & 0 \\ -\frac{4}{3} & -\frac{1}{3} & -\frac{4}{3} & 0 & 1 & 0 \\ 0 & -2 & -3 & 0 & 0 & 1 \end{array}\right]$$

6.1. $\left(\begin{array}{ccc|c} 2 & -3 & 1 & 0 \\ 3 & 4 & -2 & 5 \\ 5 & 1 & -1 & \lambda \end{array}\right) \overset{\text{Gauß}}{\Longrightarrow} \left(\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 0 & -2 & 7 & 5 \\ 0 & 0 & 0 & (\lambda - 5) \end{array}\right)$ für $\lambda = 5$ lösbar

6.2. $\left(\begin{array}{ccc|c} 2 & -3 & 1 & 0 \\ 6 & 8 & -4 & 10 \\ 10 & 2 & 2\lambda & 2 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 2 & -3 & 1 & 0 \\ 0 & 17 & -7 & 10 \\ 0 & 0 & 2\lambda-2 & -8 \end{array}\right)$ $2\lambda + 2 \neq 0 \rightarrow$ lösbar

für $\lambda \neq -1$ lösbar

7. $a_1 = (1,1,1,1)$, $a_2 = (1,1,0,0)$, $a_3 = (1,0,1,0)$, $a_4 = (1,0,0,1)$

Rang der Matrix aus den vier Vektoren ist 4 (Gauß.) $\rightarrow$ Basis

$b_1 := a_1 = (1,1,1,1)$, $\quad b_2 = a_2 - \left(\frac{a_2 \cdot b_1}{b_1 \cdot b_1}\right) b_1 = \left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right)$

$b_3 = a_3 - \left(\frac{a_3 \cdot b_2}{b_2 \cdot b_2}\right) b_2 - \left(\frac{a_3 \cdot b_1}{b_1 \cdot b_1}\right) b_1 = \left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right)$

$b_4 = a_4 - \left(\frac{a_4 \cdot b_3}{b_3 \cdot b_3}\right) b_3 - \left(\frac{a_4 \cdot b_2}{b_2 \cdot b_2}\right) b_2 - \left(\frac{a_4 \cdot b_1}{b_1 \cdot b_1}\right) b_1 = \left(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right)$

$b_1^\circ = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$, $\quad b_2^\circ = b_2$, $\quad b_3^\circ = b_3$, $\quad b_4^\circ = b_4$