1.1. $b_1 = (0, 1, 1)$, $b_2 = (1, -\frac{1}{2}, \frac{1}{2})$

$b_1^\circ = \frac{1}{2}\sqrt{2}\,(0,1,1)$ $\quad b_2^\circ = \frac{\sqrt{2}}{\sqrt{3}}\,(1, -\frac{1}{2}, \frac{1}{2})$

1.2. $r^\perp \cdot a_1 = 0$, $r^\perp \cdot a_2 = 0$ $\to$ GLS $\to$ $r^\perp = t\,(-1,-1,1)$

1.3. $a = (1,1,0)$; $p_a^{b_1^\circ} = (a \cdot b_1^\circ)\, b_1^\circ$, $p_a^{b_2^\circ} = (a \cdot b_2^\circ)\, b_2^\circ$

$\quad (0, \frac{1}{2}, \frac{1}{2}) \qquad (\frac{1}{3}, -\frac{1}{6}, \frac{1}{6})$

$$p_a^U = p_a^{b_1^\circ} + p_a^{b_2^\circ} = \underline{(\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{2}{3})} \quad (*)$$

1.4. $a + r^\perp \cdot U$: $\underbrace{\begin{pmatrix}1\\1\\0\end{pmatrix} + s\begin{pmatrix}-1\\-1\\1\end{pmatrix}}_{F} = t\begin{pmatrix}0\\1\\1\end{pmatrix} + v\begin{pmatrix}1\\0\\1\end{pmatrix}$:

s	t	v	
1	0	1	1
1	1	0	1
-1	1	1	0

$v = \frac{1}{3}$, $t = \frac{1}{3}$, $s = \frac{2}{3}$ $\to$ $F = (\frac{1}{3}, \frac{1}{3}, \frac{2}{3})$ siehe $(*)$

1.5. $d(a, U) = |F - a| = |\underbrace{a + \tfrac{2}{3}(-1,-1,1)}_{F} - a| = \frac{2}{3}\sqrt{3}$

usch

1.6. $p_a^{U^\perp} = r^\perp \cdot a = \dfrac{\begin{pmatrix}-1\\-1\\1\end{pmatrix}\cdot\begin{pmatrix}1\\1\\0\end{pmatrix}}{3} \cdot \begin{pmatrix}-1\\-1\\1\end{pmatrix} = -\frac{2}{3}\begin{pmatrix}-1\\-1\\0\end{pmatrix}$ $\Rightarrow$ $a = p_a^{U^\perp} + p_a^U$

2.1. $r^\perp = \begin{pmatrix}x\\y\\z\\v\end{pmatrix}$. $r^\perp \cdot a = 0$, $r^\perp b = 0$

x	y	z	v	
1	0	1	1	0
1	1	0	0	0

$$r^\perp = s\begin{pmatrix}-1\\1\\1\\0\end{pmatrix} + t\begin{pmatrix}-1\\1\\0\\1\end{pmatrix}$$

2.2. F : $\underbrace{\begin{pmatrix}0\\1\\0\\2\end{pmatrix}}_{P} + \underbrace{s\begin{pmatrix}-1\\1\\1\\0\end{pmatrix} + t\begin{pmatrix}-1\\1\\0\\1\end{pmatrix}}_{U^\perp} = \underbrace{v\begin{pmatrix}1\\0\\1\\1\end{pmatrix} + w\begin{pmatrix}1\\1\\0\\0\end{pmatrix}}_{U}$

s	t	v	w	
1	1	1	1	0
-1	-1	0	1	1
-1	0	1	0	0
0	-1	1	0	2

$\Rightarrow$

1	1	1	1	0
0	1	2	1	0
0	0	1	2	1
0	0	0	-5	-1

$(s, t, v, w) = (\frac{3}{5}, -\frac{7}{5}, \frac{3}{5}, \frac{1}{5})$

$$F = \begin{pmatrix}0\\1\\0\\2\end{pmatrix} + \frac{3}{5}\begin{pmatrix}-1\\1\\1\\0\end{pmatrix} - \frac{7}{5}\begin{pmatrix}-1\\1\\0\\1\end{pmatrix} = \underline{(\tfrac{4}{5}, \tfrac{1}{5}, \tfrac{3}{5}, \tfrac{3}{5})}$$

$\vartheta = F - P = (\frac{4}{5}, -\frac{4}{5}, \frac{3}{5}, -\frac{7}{5})$ $\quad |\vartheta| = \frac{1}{5}\sqrt{90}$

Abstand: $\vartheta = F - P = (\frac{4}{5}, -\frac{4}{5}, \frac{3}{5}, -\frac{7}{5})$, $|\vartheta| = \frac{1}{5}\sqrt{90}$

Spiegelung im $\mathbb{R}^2$

3. $\boxed{\mathbb{R}^2}$ $\quad A = (7, -1)$; $\quad g_0: \ \vec{x} = \lambda \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$g_1: \ \vec{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

3.1. $\vec{x} = \begin{pmatrix} 7 \\ -1 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

3.2. A an g_0 spiegeln $\to A_0$;

$$p(A, g_0) = \frac{A \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}}{\begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}; \quad A_0 = 2\,p(A, g_0) - A = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$$

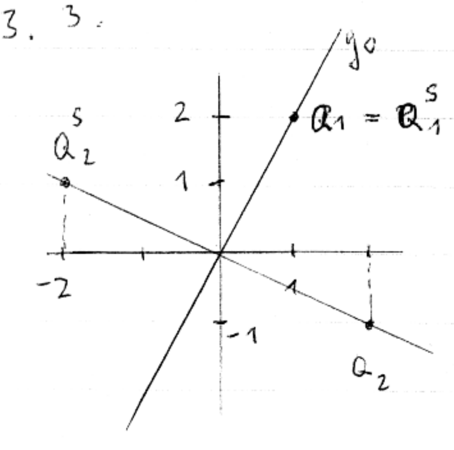
A an g_1 spiegeln $\to A_1$;

($p(A, g_1)$ ist Projektion von $\left[A - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]$ auf g_1)

$$p(A, g_1) = \frac{\left[A - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}}{\begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{4}{5} \\ \frac{8}{5} \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + p - \left[A - \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} + p\right]\right] = 2p + 2\begin{pmatrix} 1 \\ 0 \end{pmatrix} - A = \begin{pmatrix} -\frac{17}{5} \\ \frac{21}{5} \end{pmatrix}$$

3.3.



$$Q_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \xrightarrow{S} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = Q_1^S$$

$$Q_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \xrightarrow{S} \begin{pmatrix} -2 \\ 1 \end{pmatrix} = Q_2^S$$

$$\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} = S_0 \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$$

$\Downarrow$

$$S_0 = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}^{-1}$$

$\Downarrow$

$$\begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{pmatrix}$$

$$S_0 = \begin{pmatrix} -\frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix}$$

$\Leftarrow$

allgemein:

P an g_0 gespiegelt

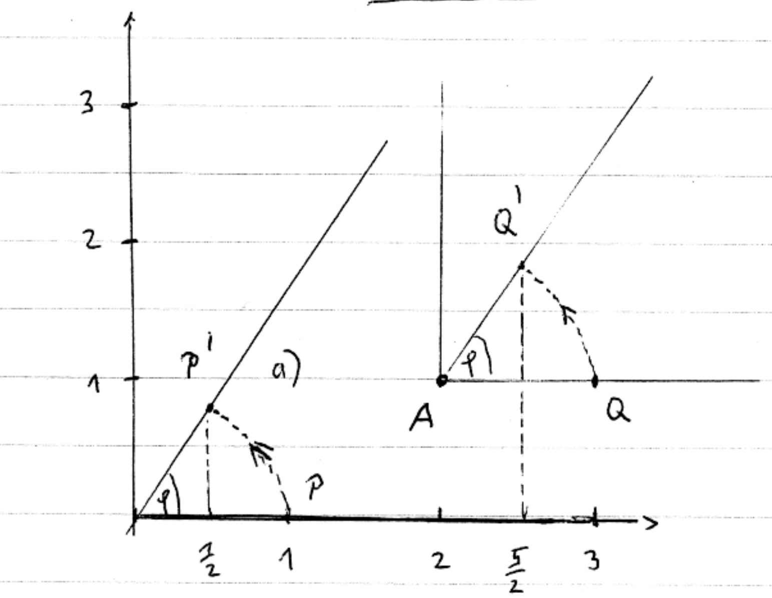
$P_0 = S_0 \cdot P$

P an g_1 gespiegelt

$P_1 = S_0\left(P - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right) + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Transformation in Ursprung, dann wieder zurück!

4. <u>Drehung mit Drehmatrix</u>



Drehmatrix

$$D = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$$

$$P' = D \cdot P$$

falls Drehpunkt $\underline{(0,0)}$

$$P(A,\varphi) = D(P-A) + A$$

falls Drehpunkt in A.

Beispiele: a) $\varphi = 60^\circ$, $A = (0,0)$

$$P = (1,0)^T = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad D_{60} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$$

$$P((0,0),60^\circ) = P' = D_{60} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2}\sqrt{3} \end{pmatrix}$$

b) $\varphi = 60^\circ$, $A = (2,1)$, D wie bei a)

$$Q = (3,1)^T$$

$$Q' = Q(A,60^\circ) = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}\left[\begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix}\right] + \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{5}{2} \\ 1 + \frac{1}{2}\sqrt{3} \end{pmatrix}.$$

zu 4.2. $A(B, 60^\circ) = D_{60} \cdot (A - B) + B$

$B(A, 45^\circ) = D_{45}(B - A) + A \qquad D_{45} = \begin{pmatrix} \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix}$