Hardwaregrundlagen

für den Studiengang
Informatik

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Mathematische Voraussetzungen:

Elementarmathematik

lineare Algebra

Lösung linearer Gleichungssysteme
(elementare Lösungsmethoden)

Integral- und Differentialrechnung

Differenzieren elementarer Funktionen
Grundintegrale
bestimmte und unbestimmte Integrale

komplexe Rechnung

Darstellungsformen komplexer Zahlen in der Gaußschen Zahlenebene
Grundrechenarten mit komplexen Zahlen

0 Einleitung

1 Grundgrößen und Grundelemente der Elektrotechnik und Elektronik

Ladung, Strom, Energie, Spannung, Leistung und Wirkungsgrad

Passive und aktive Bauelemente der Elektrotechnik

Grundstromkreis und stationäre Vorgänge in Gleichstromkreisen

Energiespeicherelemente der Elektrotechnik

2 Die Übertragung von Informationen durch elektrische Wechselgrößen

Arten und Kenngrößen von Wechselgrößen

Die Darstellung sinusförmiger Wechselgrößen in der Gaußschen Zahlenebene

Wechselstromschaltungen und Systeme

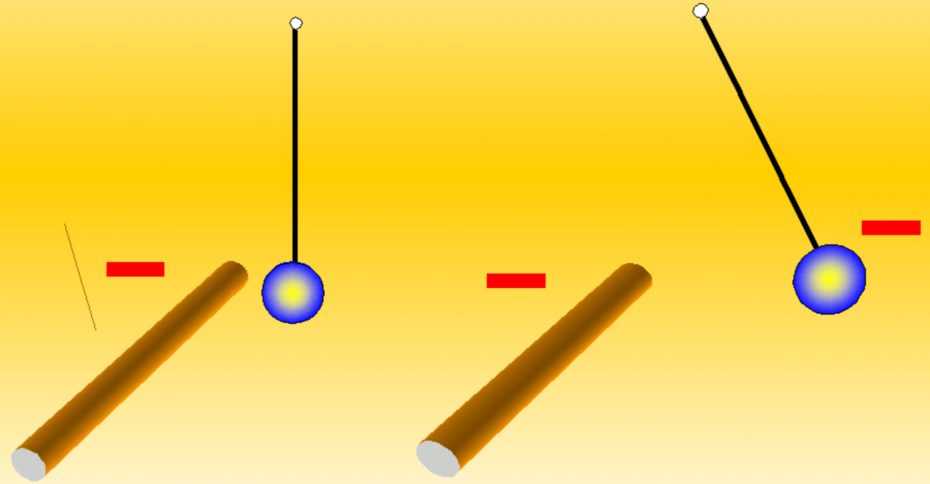
Literaturempfehlung

Seidel, Wagner — Allgemeine Elektrotechnik Band 1
1999 Carl Hanser Verlag München Wien

Seidel, Wagner — Allgemeine Elektrotechnik Band 2
2000 Carl Hanser Verlag München Wien

1. Grundgrößen und Grundelemente der Elektrotechnik / Elektronik

1.1 Die elektrische Ladung Q,q



Eigenschaften der Ladung:

- Ladungen können positive und negative Polarität haben
  - gleichnamig geladene Körper stoßen sich ab, ungleichnamig geladene Körper ziehen sich an
    - Ladungen sind an Ladungsträger gebunden und nicht unendlich teilbar

[Q] = 1 C = 1 As (Coulomb)

$$e = 1{,}6021892 \cdot 10^{-19}\ \mathrm{C}$$

Satz von der Erhaltung der Ladung:

In einem abgeschlossenen System ist die Summe aller Ladungen konstant.

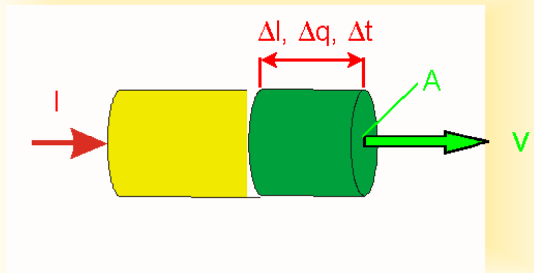
$$\sum_i Q_i = konst$$

1.2 Der elektrische Strom

In Leitermaterialien sind mobile Ladungsträger vorhanden, d. h. unter dem Einfluß eines elektrischen Feldes entsteht ein elektrischer Strom (Konvektionsstrom).

Ladungsträgerbewegungen und damit elektrische Ströme können auch durch den Ausgleich unterschiedlicher Konzentrationen von Ladungen durch Diffusionsvorgänge entstehen (Diffusionsströme).

- die Stromstärke I, i



$$i = \frac{\Delta q}{\Delta t}$$

$$i = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$$

Die Einheit der elektrischen Stromstärke ist das Ampere

$$i = \frac{dq}{dt}$$

- Die positive Stromrichtung ist die Strömungsrichtung der positiven Ladungsträger.
- Elektrische Ströme, gleich welcher Art ,sind immer von einem Magnetfeld umgeben.
- Konvektionsströme sind mit **Stofftransport** und **Wärmeentwicklung** verbunden.

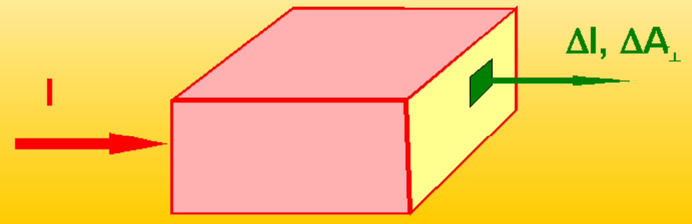
umgekehrt wird die Ladung:

$$q(t) = Q_0 + \int_{t_0}^{t} i(\tau)\, d\tau$$

und einfacher für Gleichstrom:

$$q(t) = Q_0 + I\,(t - t_0)$$

Die Verteilung des Stromes in Leitern, die elektrische Stromdichte **J**

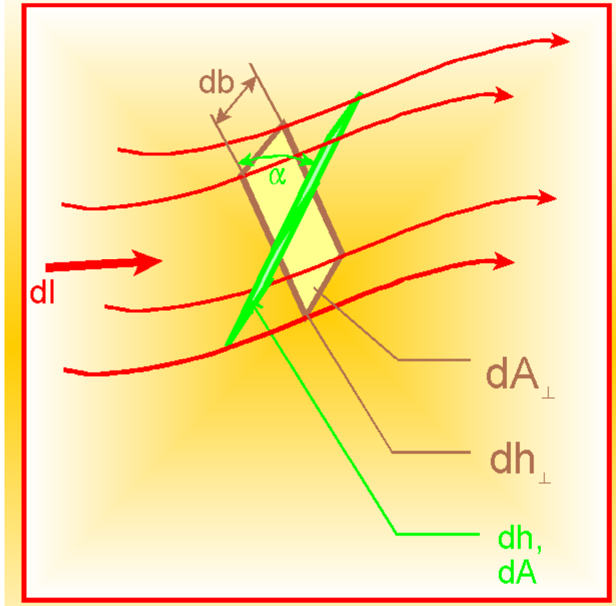


für Betrag und Einheit der Stromdichte ergeben sich:

$$J = \lim_{\Delta A_\perp \to 0} \frac{\Delta I}{\Delta A_\perp} = \frac{dI}{dA_\perp}$$

$$I = \int_{A_\perp} J\, dA_\perp$$

[J] = 1A/mm²



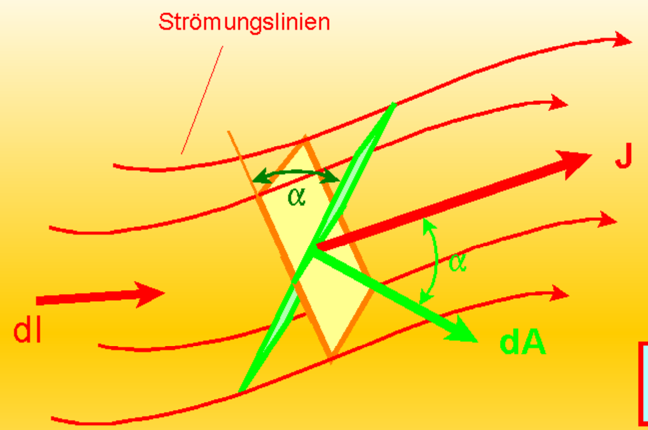
$$dI = J\, dA_\perp$$

$$dA_\perp = db\, dh_\perp$$

$$dA = db\, dh$$

$$dh_\perp = dh \cos\alpha$$

$$dA_\perp = dA \cos\alpha$$



$$dI = J\, dA_\perp$$

$$dA_\perp = dA \cos\alpha$$

$$dI = J\, dA \cos\alpha$$

$$I = \int_A \vec{J}\, d\vec{A}$$

$$dI = \vec{J}\, d\vec{A}$$

Knotensatz (1. Kirchhoffscher Satz)

$$\sum_i Q_i = konst.$$

$$\frac{d}{dt}\left(\sum_i Q_i\right) = 0$$

$$\sum_i I_{i\,vorzeichen} = 0$$

oder

$$\sum_i I_{i\uparrow} = \sum_i I_{i\downarrow}$$

Verallgemeinerter Knotensatz

$$\sum_i I_{i\,vorzeichen} = 0$$

$$I = \int_A \vec{J}\, d\vec{A}$$

$$\oint \vec{J}\, d\vec{A} = 0$$

1.3 Feldstärke, Kraft, Energie, Spannung, Leistung und Wirkungsgrad

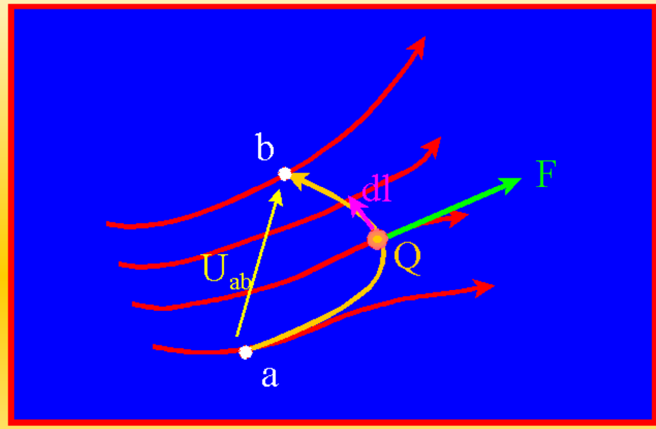
$$\vec{F} = Q\,\vec{E}$$

elektrische Feldstärke

$$\vec{E} = \frac{\vec{F}}{Q}$$

$$[E] = 1\,\frac{\mathrm{V}}{\mathrm{m}}$$

die elektrische Spannung:

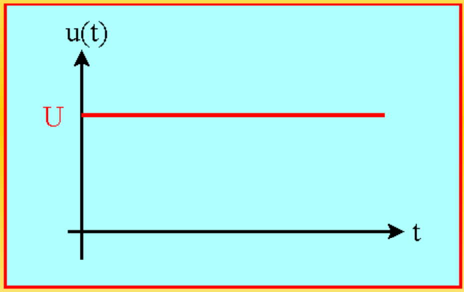


$$\Delta W_{el} = \int_a^b \vec{F}\, d\vec{l} = \int_a^b Q\,\vec{E}\, d\vec{l} = Q\int_a^b \vec{E}\, d\vec{l}$$

$$\frac{\Delta W_{el}}{Q} = \int_a^b \vec{E}\, d\vec{l} = U_{ab}$$

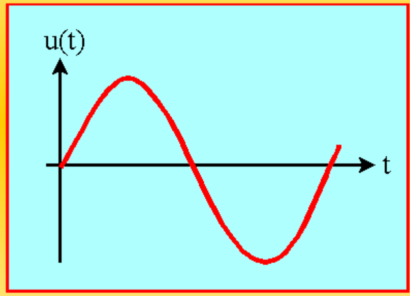
Spannungen können ebenso wie Ströme in unterschiedlicher Weise von der Zeit abhängen

$$u(t) = U$$



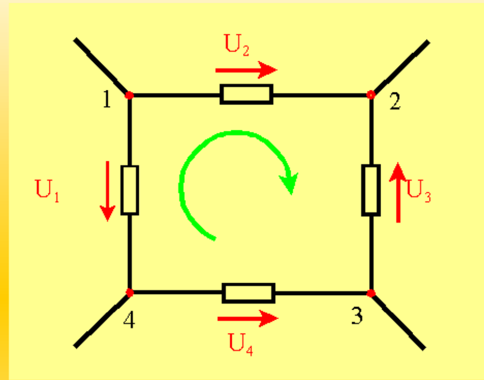
- Gleichspannung

$$u(t) = \hat{U}\sin(\omega t + \varphi_u)$$



- Wechselspannung

der Maschensatz (2. Kirchhoffscher Satz)



$$\Delta W = -QU_1 + QU_2 - QU_3 - QU_4 = 0$$

$$-U_1 + U_2 - U_3 - U_4 = 0$$

für jede Masche gilt: $$\sum_i U_{i\,vorz} = 0$$

- die elektrische Leistung

$$W_{el} = Q\,U$$

$$P_{el} = \frac{dW_{el}}{dt} = \frac{dQ}{dt}U$$

$$P_{el} = U\ I$$

mit der Einheit

[P]=[U] [I]=1VA=1W

- der Satz von der Erhaltung der Leistung

$$\sum_i W_i = konst.$$

$$\frac{d}{dt}\sum_i W_i = \sum_i \frac{dW_i}{dt} = 0$$

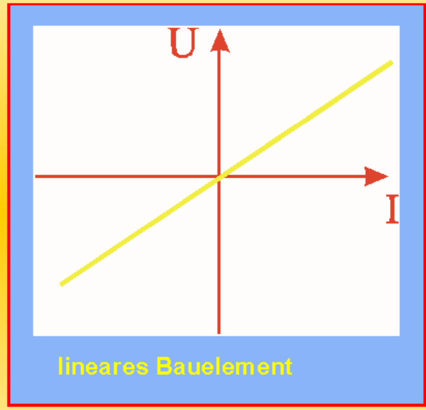
$$\sum_i P_i = 0$$

- der Wirkungsgrad

$$\eta = \frac{W_{ab}}{W_{zu}}$$

1.4 Passive Elemente der Elektrotechnik

- die Strom - Spannungs - Kennlinie



$$U = R\,I$$

- der elektrische Widerstand
(Ohmsches Gesetz)

$$R = \frac{U}{I} \quad [R] = \frac{[U]}{[I]} = 1\frac{V}{A} = 1\Omega = 1S^{-1}$$

- der elektrische Leitwert

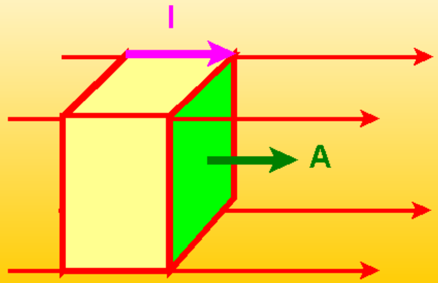
$$G = \frac{I}{U} = \frac{1}{R} \quad [G] = \frac{[I]}{[U]} = 1\frac{A}{V} = 1S = 1\Omega^{-1}$$

- die Materialgleichung

$$\vec{J} = \gamma\,\vec{E}$$

$$[\gamma] = 1\ Sm^{-1}$$

- die Widerstandsbemessungsgleichung



$$R = \frac{U}{I} = \frac{E}{J}\,\frac{l}{A}$$

$$\rho = \frac{1}{\gamma}\,. \qquad [\,\rho\,] = 1\Omega m,\ 1\frac{\Omega mm^2}{m},\ 1\Omega cm\,.$$

$$R = \frac{U}{I} = \frac{E}{J}\,\frac{l}{A} = \frac{l}{\gamma A} = \rho\frac{l}{A}$$

im homogenen Feld gilt

$$U = E\,l$$

$$I = J\,A$$

$$\rho(T) = \rho(T_o)\,(1 + \alpha_{T_o}\Delta T)$$

$$\rho(T) = \rho_{20}\,(1 + \alpha_{20}\,\Delta T + \beta_{20}\,(\Delta T)^2)$$

Beispiele für Widerstandsbauformen:

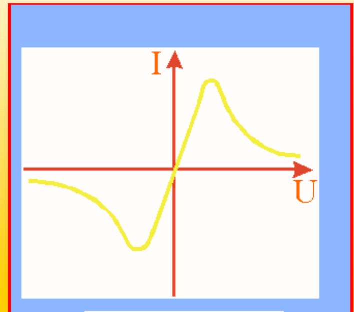
Drahtwiderstand

Metallschichtwiderstand

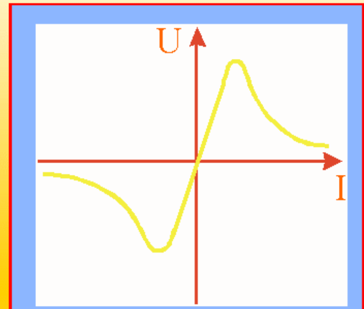
Hochlastwiderstand

Regelbarer Widerstand

Beispiele für nichtlineare passive Elemente



Thermistor (PTC-Typ)

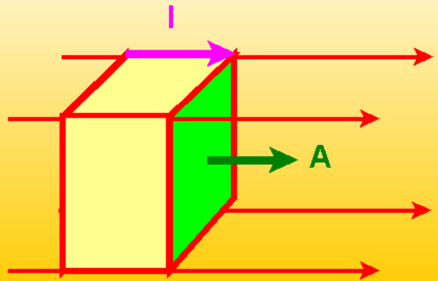


Thermistor (NTC-Typ)

PTC - Widerstand

NTC - Widerstand

- die Widerstandsbemessungsgleichung



$$R = \frac{U}{I} = \frac{E}{J}\,\frac{l}{A}$$

$$\rho = \frac{1}{\gamma}\,. \qquad [\,\rho\,] = 1\Omega m,\ 1\frac{\Omega mm^2}{m},\ 1\Omega cm\,.$$

im homogenen Feld gilt

$$U = E\,l$$

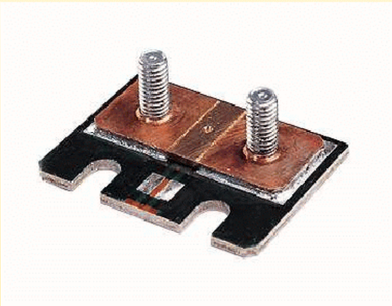
$$I = J\,A$$

$$R = \frac{U}{I} = \frac{E\,l}{J\,A} = \frac{l}{\gamma A} = \rho\frac{l}{A}$$

$$\rho(T) = \rho(T_o)\,(1 + \alpha_{T_o}\Delta T)$$

$$\rho(T) = \rho_{20}\,(1 + \alpha_{20}\,\Delta T + \beta_{20}\,(\,\Delta T\,)^2\,)$$

Hochlastwiderstand



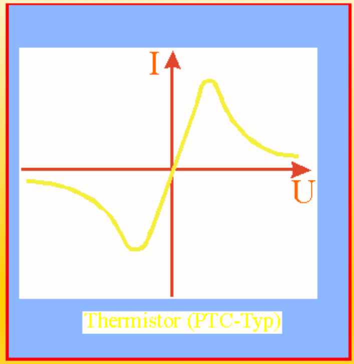
Regelbarer Widerstand

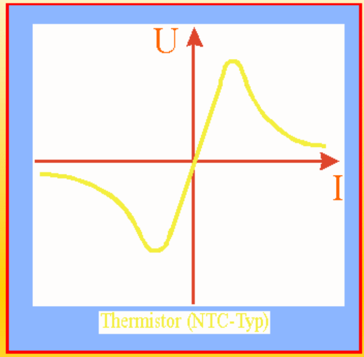
Beispiele für Widerstandsbauformen:

Drahtwiderstand

Metallschichtwiderstand

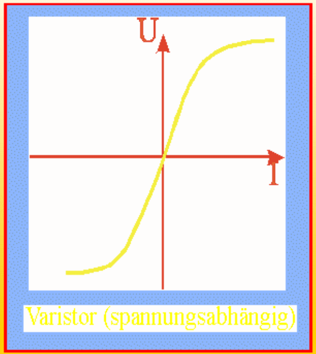
Beispiele für nichtlineare passive Elemente





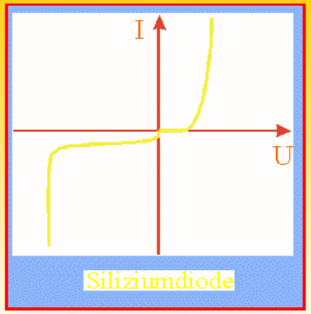
PTC - Widerstand

NTC - Widerstand



Spannungsabhängiger Widerstand

$$\boxed{I = K\,U^{\alpha}}$$

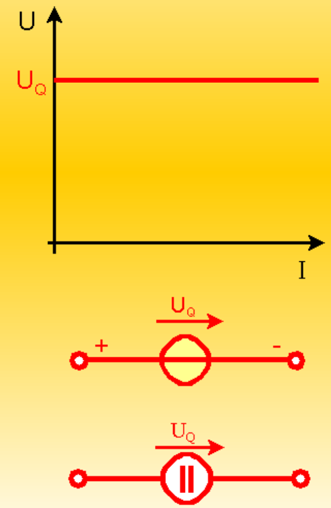


$$\boxed{I = I_S\left(e^{\frac{U}{U_T}} - 1\right)}$$

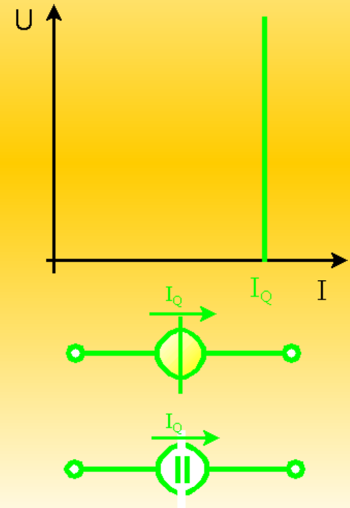
1.5 Das aktive Element

1.5.1 Ideale ungesteuerte Spannungs- und Stromquellen

- das Modell der Spannungsquelle

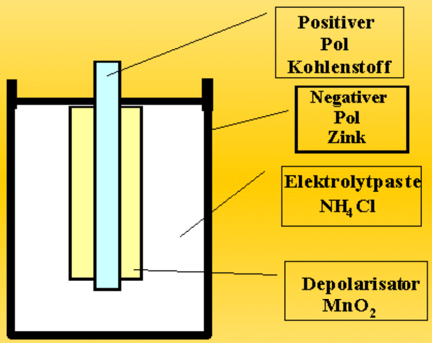


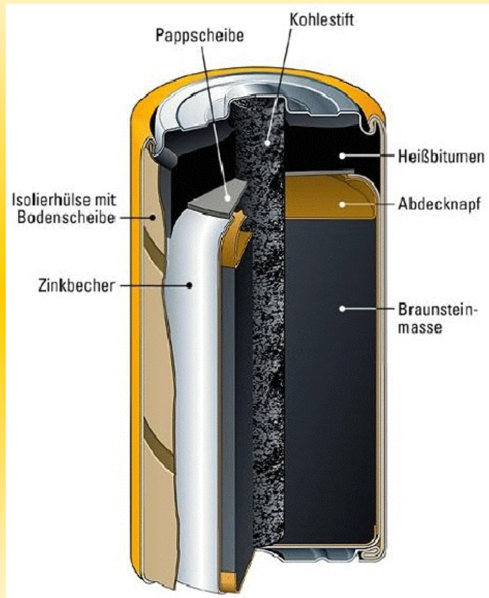
- das Modell der Stromquelle



<u>Primärelement</u>

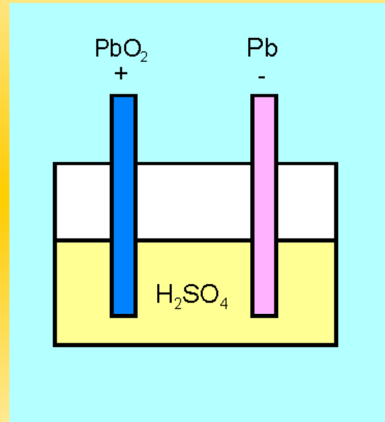
Beispiel: Zink - Kohle - Batterie



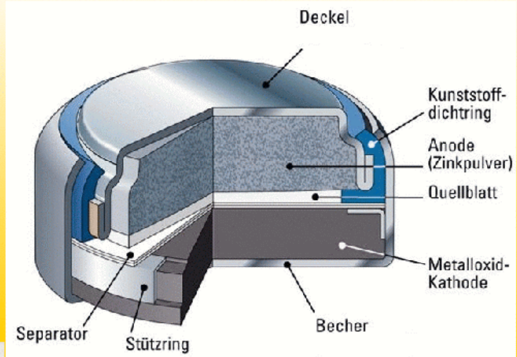


<u>Sekundärelement</u>

Beispiel: Blei-Akkumulator



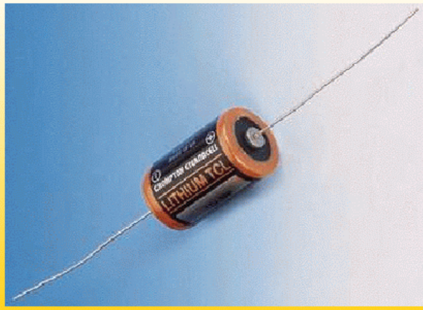
Knopfzelle



Alkali-Mangan-Knopfzelle

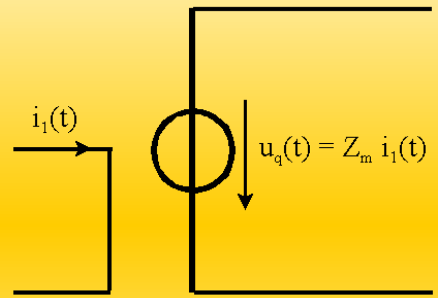
Alkalibatterie

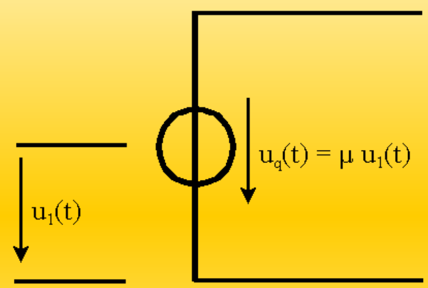
Speicherbatterie



Lithiumbatterie

1.5.2 Ideale gesteuerte Spannungs- und Stromquellen



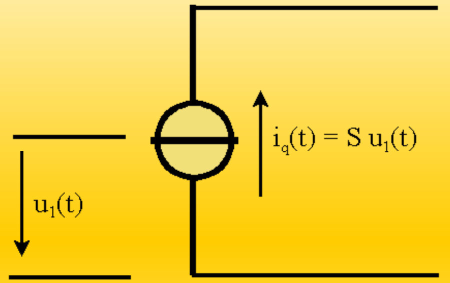


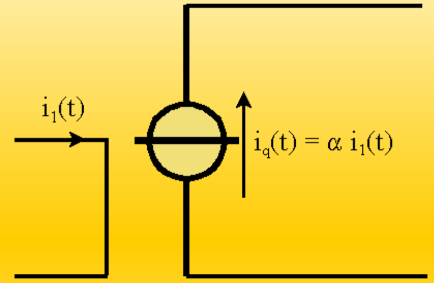
stromgesteuerte Spannungsquelle

Z_m - Transferwiderstand

spannungsgesteuerte Spannungsquelle

μ -Leerlaufspannungsverstärkung





spannungsgesteuerte Stromquelle

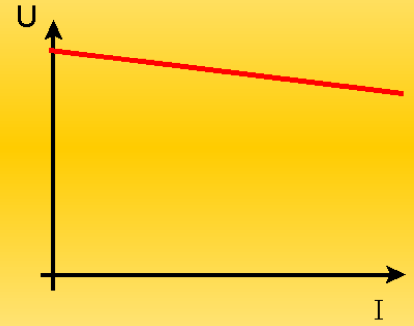
S - Steilheit

stromgesteuerte Stromquelle

α - Kurzschlußstromverstärkung

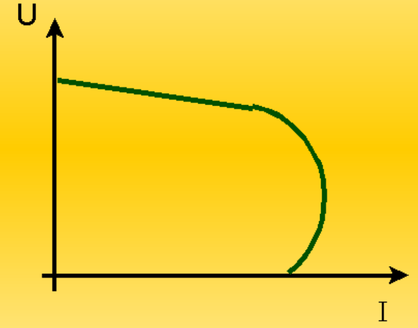
1.5.3 Die Strom-Spannungs-Kennlinie realer aktiver Elemente

linearer Fall



Akkumulator

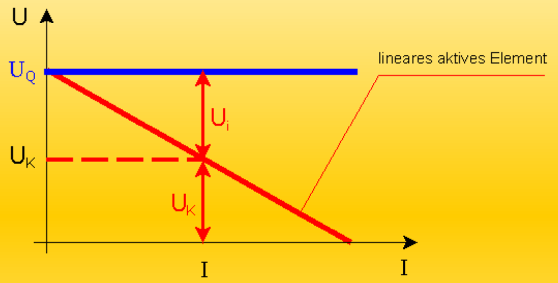
nichtlinearer Fall

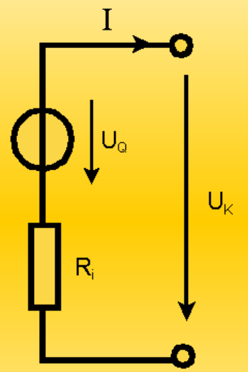


Gleichstromnebenschlußgenerator

- die Modellierung des elektrischen Verhaltens eines realen linearen aktiven Elementes durch Ersatzschaltbilder

das Spannungsquellenersatzschaltbild



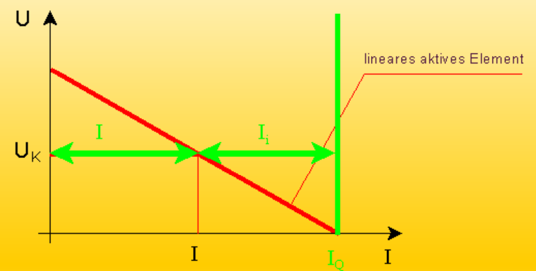


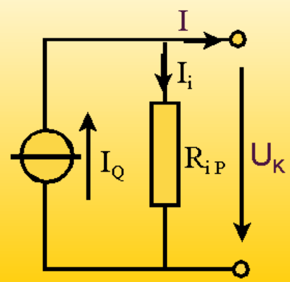
$$U_K = U_Q - U_i$$

$$U_i = R_i\, I$$

$$U_K = U_Q - R_i\, I$$

das Stromquellenersatzschaltbild





$$I = I_Q - I_i$$

$$I_i = \frac{U_K}{R_{i\,P}}$$

$$I = I_Q - \frac{U_K}{R_{i\,P}}$$

- die Äquivalenz von Spannungs- und Stromquellenersatzschaltbild

Spannungsquelle:

$$U_K = U_Q - R_i\, I$$

Stromquelle

$$I = I_Q - \frac{U_K}{R_{iP}}$$

$$U_K = I_Q\, R_{iP} - R_{iP}\, I$$

damit erhält man die Beziehungen

$$U_Q = I_Q\, R_i$$

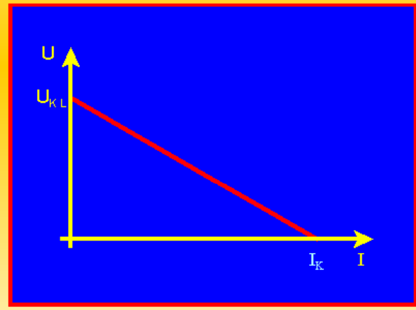
$$R_i = R_{iP}$$

- die extremen Betriebszustände aktiver Elemente

$$U_K = U_Q - R_i\, I \qquad I = I_Q - \frac{U_K}{R_{iP}}$$

1. Leerlauf (I=0):

$$U_{KL} = U_Q$$

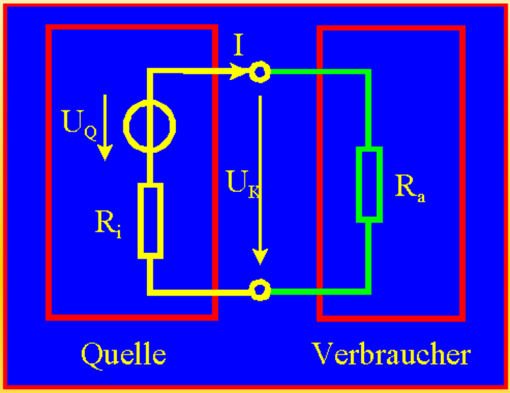


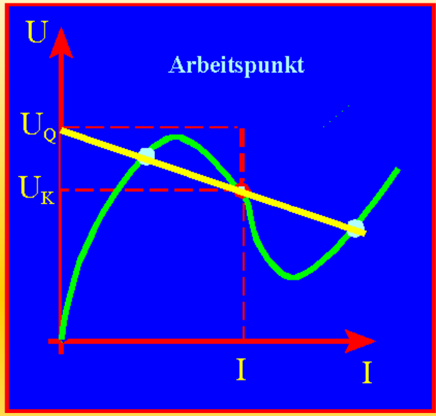
2. Kurzschluß (U_K=0):

$$I_K = I_Q = \frac{U_Q}{R_i}$$

1.6 Der Grundstromkreis und stationäre Vorgänge in Gleichstromschaltungen

1.61 Der Grundstromkreis

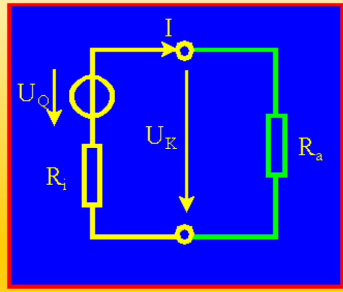




$$I_{Quelle} = I_{Verbraucher}$$

$$U_{K\,Quelle} = U_{K\,Verbraucher}$$

- rechnerische Lösung



$$U_{K\,Quelle} = U_Q - R_i\, I = U_{K\,Verbraucher} = I\, R_a$$

$$I = \frac{U_Q}{(R_i + R_a)} \qquad U_K = \frac{R_a\, U_Q}{(R_i + R_a)}$$

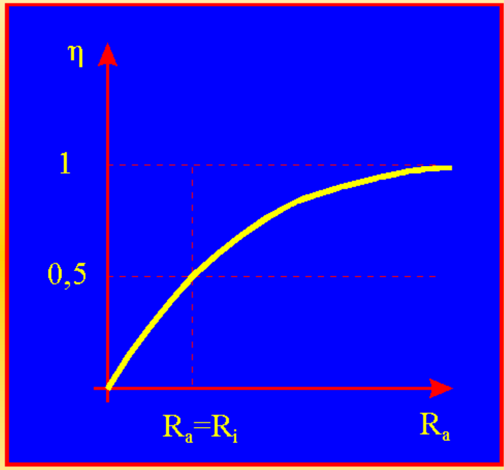
$$P_a = I\, U_K = \frac{R_a\, U_Q^2}{(R_i + R_a)^2} \qquad \eta = \frac{P_a}{P_Q} = \frac{U_K I}{U_Q I} = \frac{R_a}{R_i + R_a}$$

- Kurvendiskussion

$$\eta = \frac{P_a}{P_Q} = \frac{U_K I}{U_Q I} = \frac{R_a}{R_i + R_a}$$

$$\eta(R_a = 0) = 0$$

$$\eta(R_a \to \infty) = 1$$



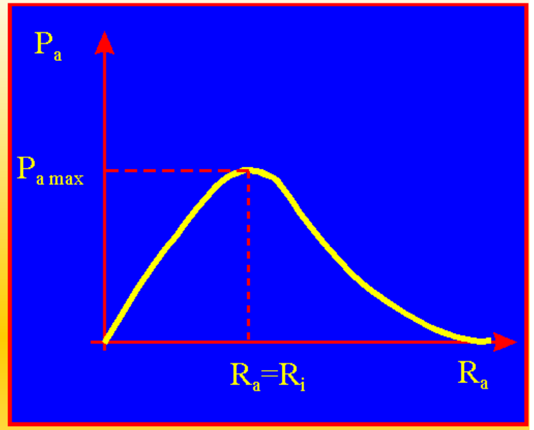
$$P_a = I\,U_K = \frac{R_a U_Q^2}{(R_i + R_a)^2}$$

$$P_a(R_a = 0) = 0$$

$$P_a(R_a \to \infty) = 0$$

$$\frac{dP_a}{dR_a} = \frac{(R_i + R_a)^2 - 2R_a(R_i + R_a)}{(R_i + R_a)^4} U_Q^2 = 0$$

$$(R_i + R_a) - 2R_a = 0$$

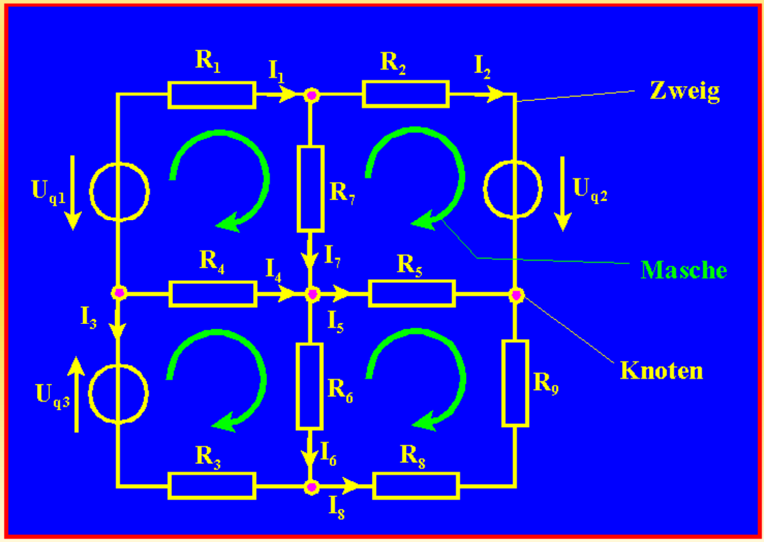


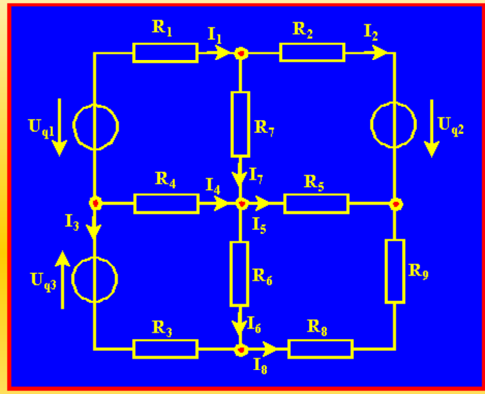
Leistungsanpassung

$$R_a = R_i$$

1.6.2 Die Berechnung von Gleichstromkreisen mittels der Kirchhoffschen Sätze

- **Grundbegriffe**





Knotensatz:

$$K1: \quad I_1 - I_2 - I_7 = 0$$

$$K2: \quad -I_1 - I_3 - I_4 = 0$$

$$K3: \quad I_2 + I_5 + I_8 = 0$$

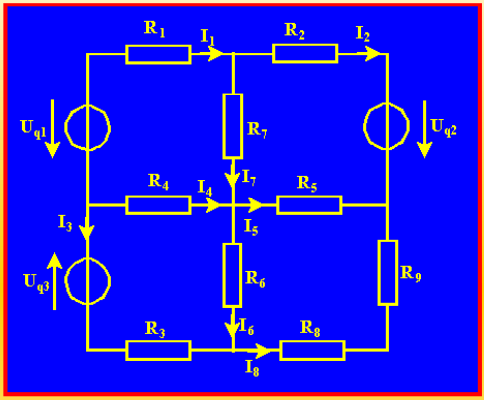
$$K4: \quad I_3 + I_6 - I_8 = 0$$

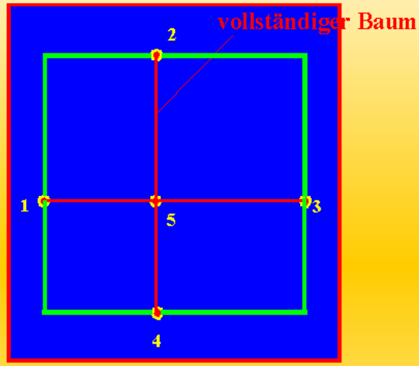
$$K5: \quad I_4 + I_7 - I_5 - I_6 = 0$$

Anzahl der linear unabhängigen Knotengleichungen:

$$\alpha = k - 1$$

Maschensatz:





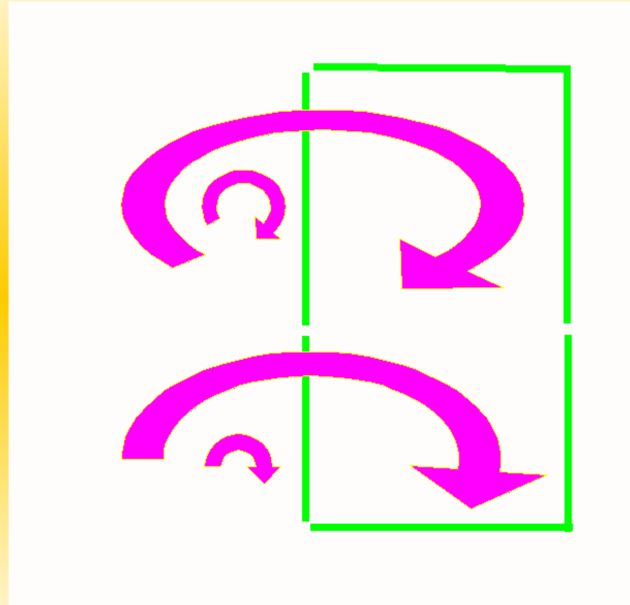
M1: $-U_{q1} + R_1 I_1 + R_7 I_7 - R_4 I_4 = 0$

M2: $U_{q2} - R_5 I_5 - R_7 I_7 + R_2 I_2 = 0$

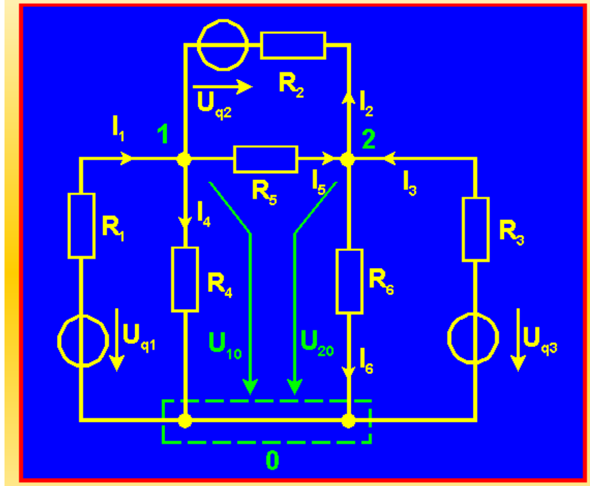
M3: $U_{q3} + R_4 I_4 + R_6 I_6 - R_3 I_3 = 0$

M4: $R_5 I_5 - (R_8 + R_9) I_8 - R_6 I_6 = 0$

Anzahl der linear unabhängigen Maschengleichungen: $\beta = z - \alpha$



1.6.3 Die Methode der Knotenspannungen (Knotenpotentiale)



$I_4 = \frac{U_{10}}{R_4}$

$I_6 = \frac{U_{20}}{R_6}$

$I_5 = \frac{U_{10} - U_{20}}{R_5}$

$I_2 = \frac{U_{20} - U_{10} + U_{q2}}{R_2}$

$I_1 = \frac{U_{q1} - U_{10}}{R_1}$

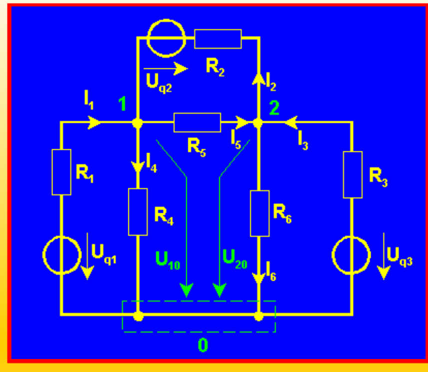
$I_3 = \frac{U_{q3} - U_{20}}{R_3}$

$-U_{10} + I_5 R_5 + U_{20} = 0$

$+U_{10} - U_{q1} + I_1 R_1 = 0$

$-U_{10} + U_{q2} - I_2 R_2 + U_{20} = 0$

$-U_{20} + U_{q3} - I_3 R_3 = 0$



$I_1 = \frac{U_{q1} - U_{10}}{R_1}$

$I_2 = \frac{U_{20} - U_{10} + U_{q2}}{R_2}$

$I_3 = \frac{U_{q3} - U_{20}}{R_3}$

$I_4 = \frac{U_{10}}{R_4}$

$I_5 = \frac{U_{10} - U_{20}}{R_5}$

$I_6 = \frac{U_{20}}{R_6}$

Knoten 1 :

$$I_1 + I_2 - I_4 - I_5 = 0$$

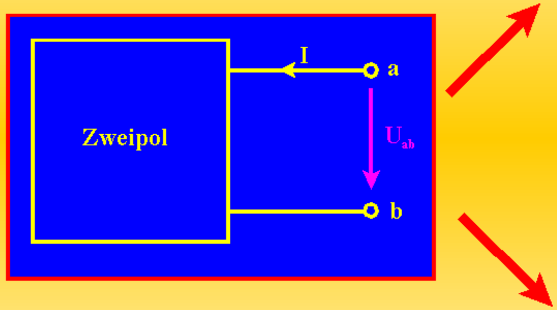
$$\frac{U_{q1} - U_{10}}{R_1} + \frac{U_{q2} + U_{20} - U_{10}}{R_2} - \frac{U_{10}}{R_4} - \frac{U_{10} - U_{20}}{R_5} = 0$$

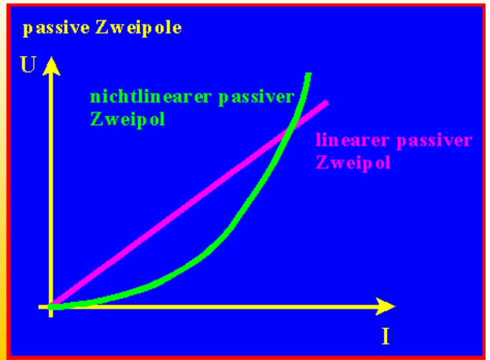
$$\frac{U_{q1}}{R_1} + \frac{U_{q2}}{R_2} = U_{10}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5}\right) - U_{20}\left(\frac{1}{R_2} + \frac{1}{R_5}\right)$$

Knoten 2:

$$-\frac{U_{q2}}{R_2} + \frac{U_{q3}}{R_3} = U_{20}\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} + \frac{1}{R_6}\right) - U_{10}\left(\frac{1}{R_2} + \frac{1}{R_5}\right)$$

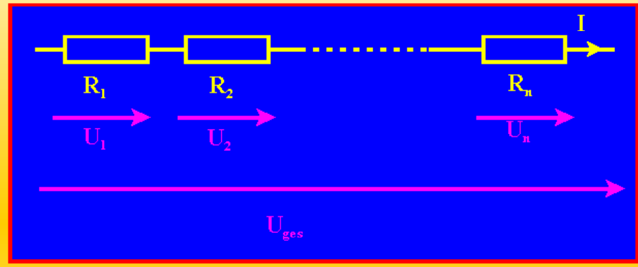
1.6.4 Die Zweipoltheorie





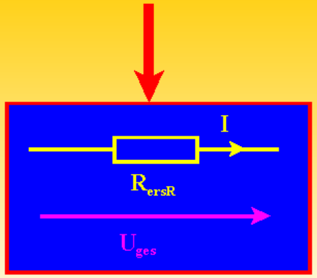
Berechnung linearer passiver Zweipole

- Reihenschaltung:



$$I_1 = I_2 = \cdots I_n = I$$

$$U_{ges} = \sum_{i=1}^{n} U_i = \sum_{i=1}^{n} R_i I = \left(\sum_{i=1}^{n} R_i \right) I$$

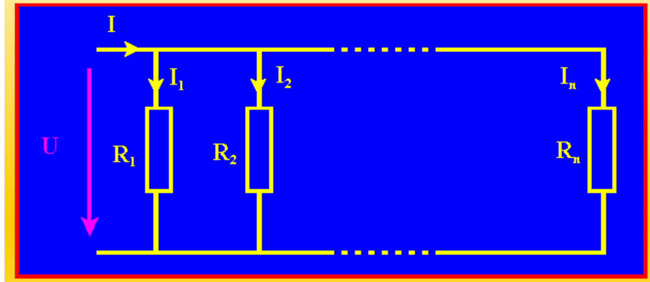


$$U_{ges} = \left(\sum_{i=1}^{n} R_i \right) I = R_{ersR} I$$

$$R_{ersR} = \sum_{i=1}^{n} R_i$$

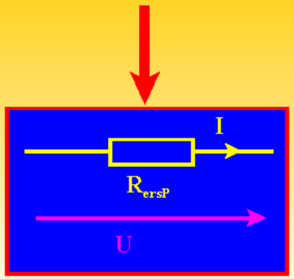
Berechnung linearer passiver Zweipole

- Parallelschaltung:



$$U_1 = U_2 = \cdots U_n = U$$

$$I = \sum_{i=1}^{n} I_i = \sum_{i=1}^{n} \frac{U}{R_i} = \left(\sum_{i=1}^{n} \frac{1}{R_i} \right) U$$

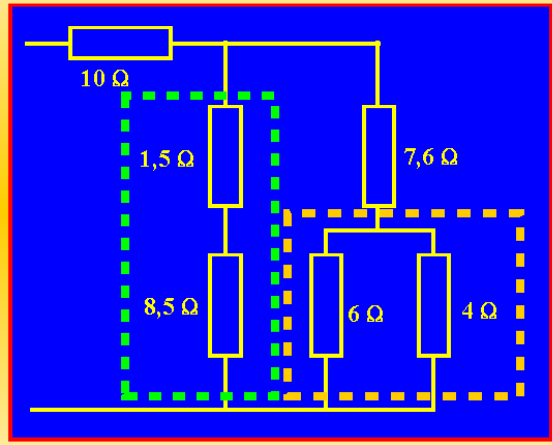


$$I = \left(\sum_{i=1}^{n} \frac{1}{R_i} \right) U = \frac{1}{R_{ersP}} U$$

$$\frac{1}{R_{ersP}} = \sum_{i=1}^{n} \frac{1}{R_i}$$

Berechnung linearer passiver Zweipole

- Reihen-Parallel-Schaltung:



$$R_{ers1} = 1{,}5\,\Omega + 8{,}5\,\Omega = 10\,\Omega$$

$$\frac{1}{R_{ers2}} = \frac{1}{6\,\Omega} + \frac{1}{4\,\Omega}$$

$$\frac{1}{R_{ers2}} = \frac{6\,\Omega + 4\,\Omega}{6\,\Omega \cdot 4\,\Omega}$$

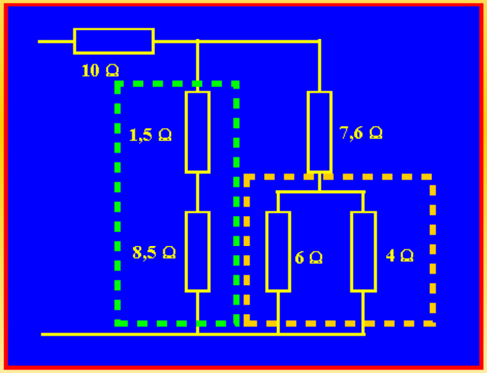
$$R_{ers2} = \frac{6\,\Omega \cdot 4\,\Omega}{6\,\Omega + 4\,\Omega} = 2{,}4\,\Omega$$

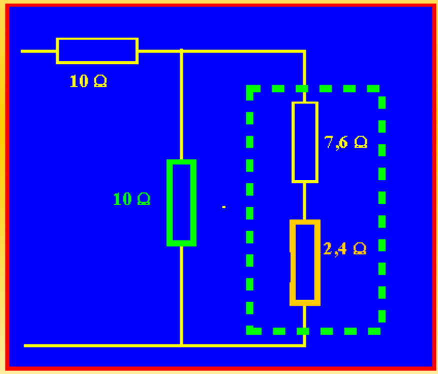
Berechnung linearer passiver Zweipole

- Reihen-Parallel-Schaltung:

$$R_{ers1} = 1{,}5\Omega + 8{,}5\Omega = 10\Omega$$

$$R_{ers3} = 2{,}4\Omega + 7{,}6\Omega = 10\Omega$$





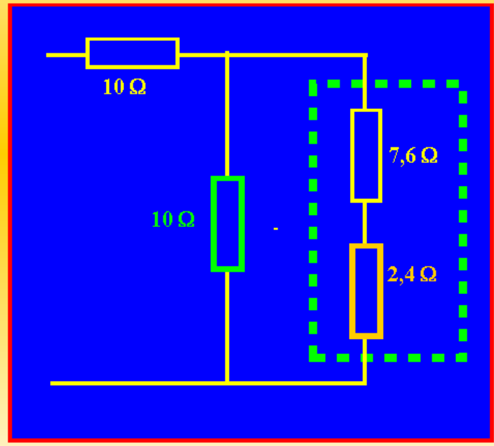
$$R_{ers2} = 2{,}4\Omega$$

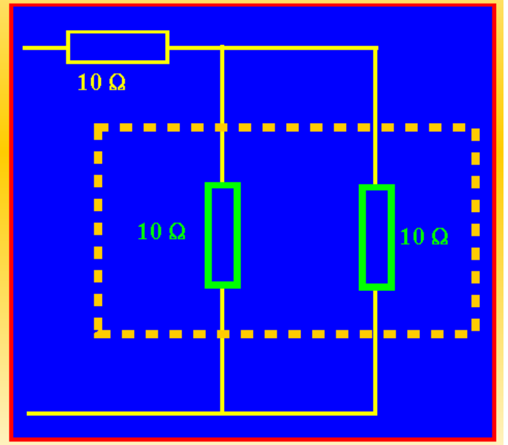
Berechnung linearer passiver Zweipole

- Reihen-Parallel-Schaltung:

$$R_{ers3} = 2{,}4\Omega + 7{,}6\Omega = 10\Omega$$

$$R_{ers4} = \frac{10\Omega \cdot 10\Omega}{10\Omega + 10\Omega} = 5\Omega$$

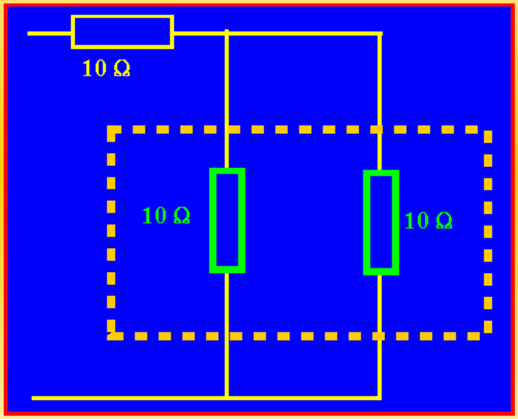


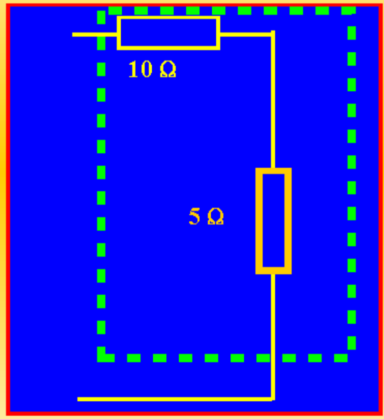


Berechnung linearer passiver Zweipole

- Reihen-Parallel-Schaltung:

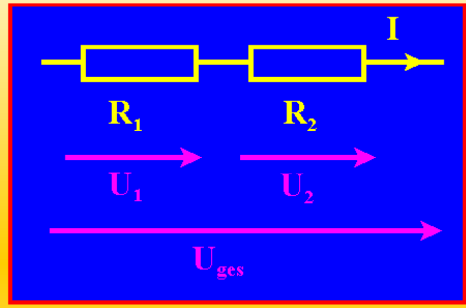
$$R_{ers} = 10\Omega + 5\Omega = 15\Omega$$





$$R_{ers4} = \frac{10\Omega \cdot 10\Omega}{10\Omega + 10\Omega} = 5\Omega$$

- Spannungsteilerregel:



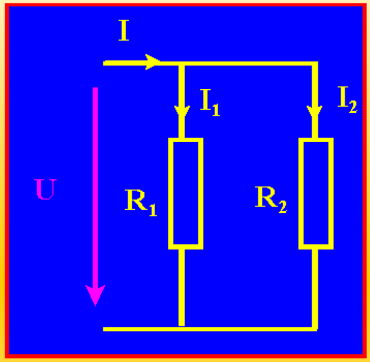
$$I_1 = I_2 = I$$

$$I_1 = \frac{U_1}{R_1} = I_2 = \frac{U_2}{R_2} = I = \frac{U_{ges}}{R_1 + R_2}$$

$$\frac{U_1}{U_2} = \frac{R_1}{R_2}$$

$$\frac{U_1}{U_{ges}} = \frac{R_1}{R_1 + R_2}$$

- **Stromteilerregel:**



$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

$$\frac{I_1}{I} = \frac{R_{ersP}}{R_1} = \frac{R_1 R_2}{R_1 (R_1 + R_2)}$$

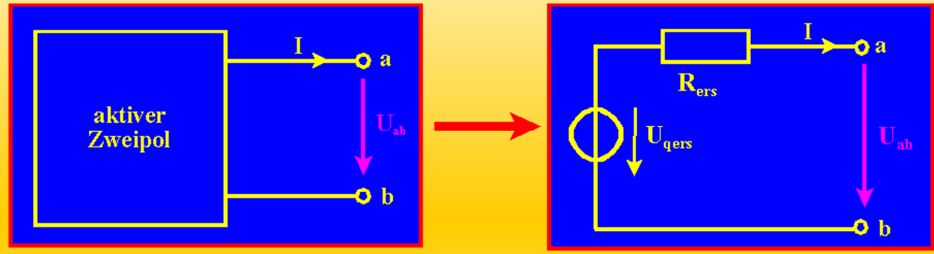
$$U_1 = U_2 = U$$

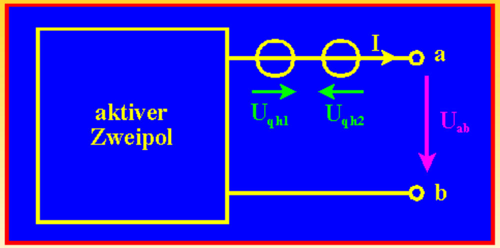
$$\frac{I_1}{I} = \frac{R_{ersP}}{R_1} = \frac{R_2}{R_1 + R_2}$$

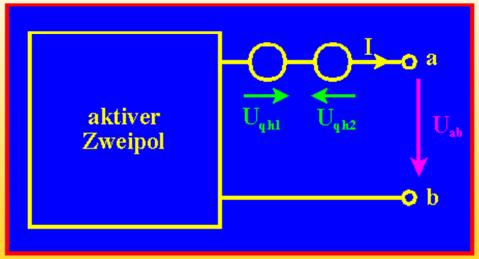
$$U_1 = I_1 R_1 = U_2 = I_2 R_2 = U = I\, R_{ersP}$$

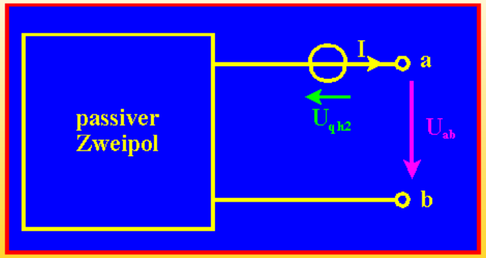
Berechnung linearer aktiver Zweipole

- Der Satz von der Ersatzspannungsquelle:





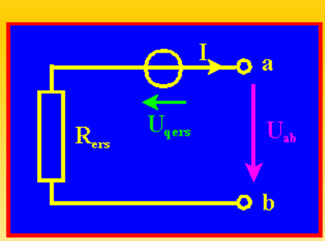




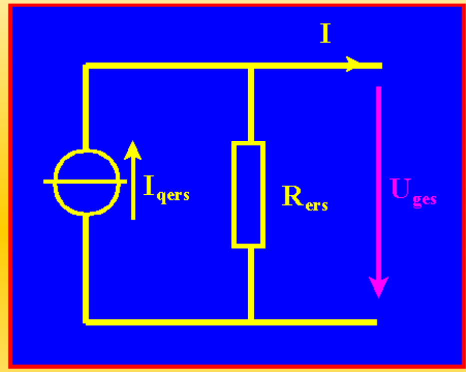
Wahl der Hilfsspannung:

1. U_{qh1} und U_{qh2} sind entgegengesetzt gleich
2. U_{qh1} wird so gewählt, daß bei ausschließlichem Vorhandensein von U_{qh1} und den inneren Strom- und Spannungsquellen im Außenkreis kein Strom fließt. Es werden also durch U_{qh1} alle aktiven Elemente im Inneren des Zweipols kompensiert.
   Es gilt also:

$$U_{qh1} = U_{abL}$$



- Der Satz von der Ersatzstromquelle:



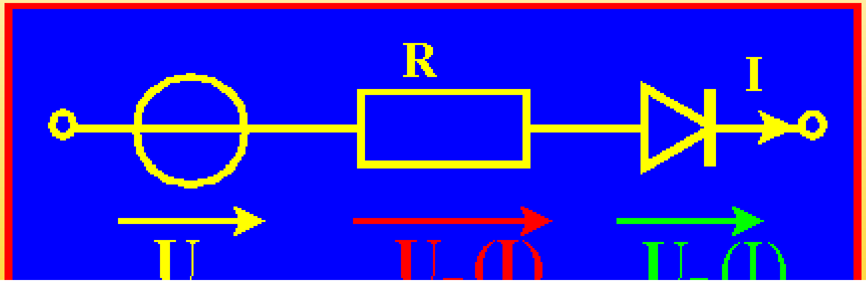
Es gilt:

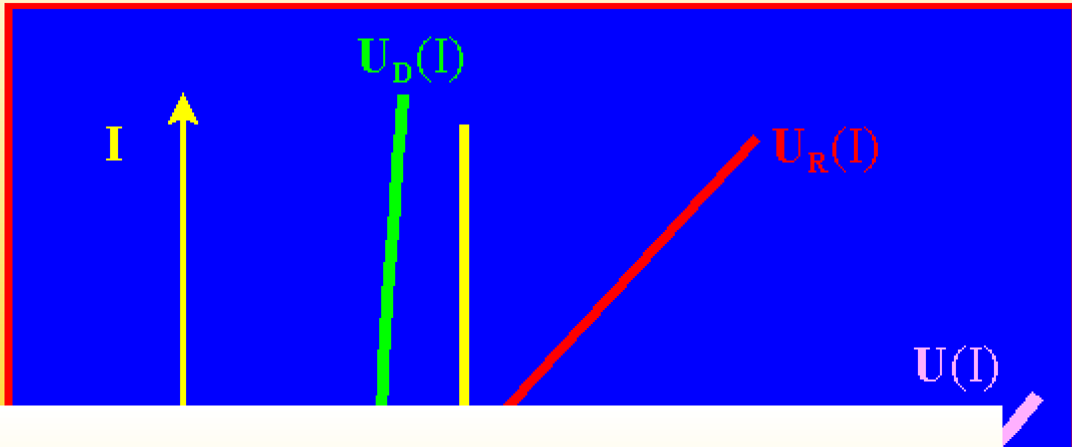
$$I_{qers} = I_{Kurzschluß}$$

$$U_{qers} = I_{qersatz}\, R_{ers}$$

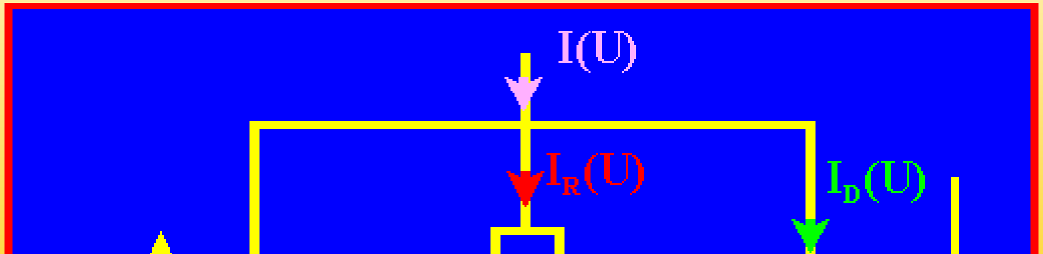
Berechnung nichtlinearer passiver Zweipole

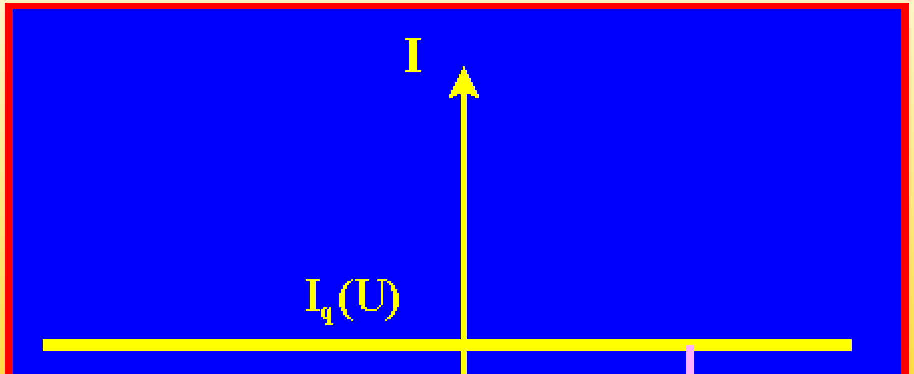
- Reihenschaltung:





- Parallelschaltung:

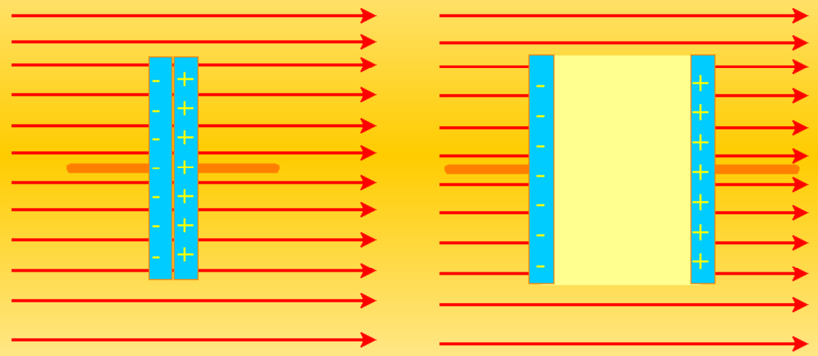




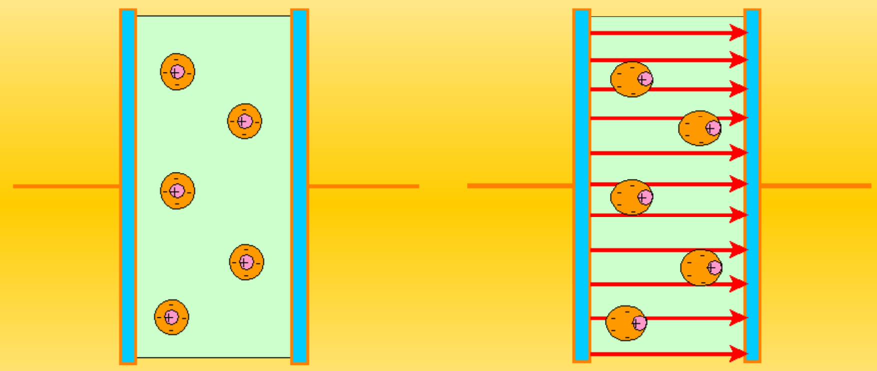
1.7 Energiespeicherelemente der Elektrotechnik

1.7.1 Kapazität und Kondensator

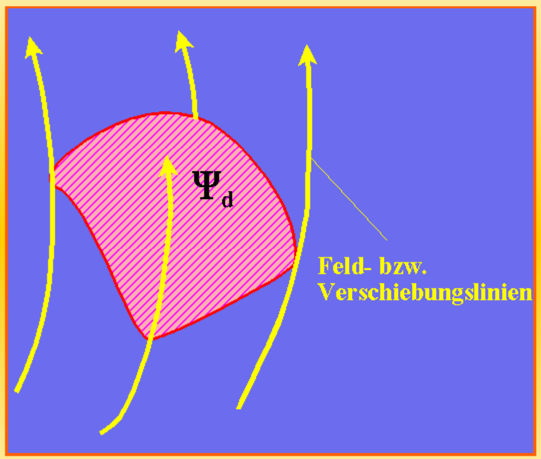
- Influenz



- Polarisation



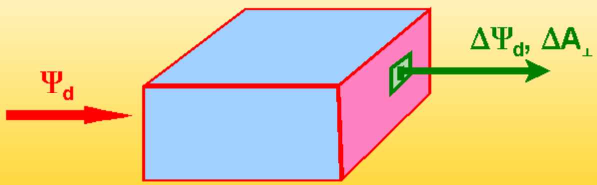
- der ektrische Fluß Ψ_d



$$\Psi_d = Q_{getrennt}$$

$$[\Psi_d] = [Q] = 1As = 1C$$

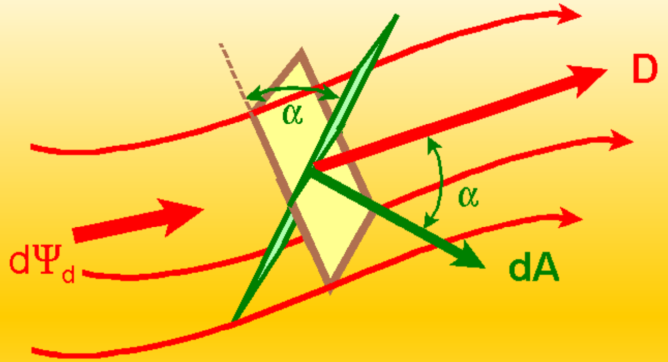
- die ektrische Flußdichte D



Betrag und Einheit der Verschiebungsflußdichte werden:

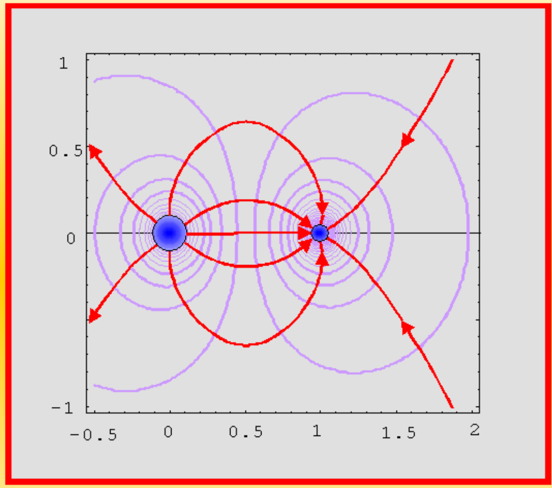
$$D = \lim_{\Delta A_\perp \to 0} \frac{\Delta\Psi_d}{\Delta A_\perp} = \frac{d\Psi_d}{dA_\perp}$$

$$[\Psi_d] = 1As/m^2$$



$$\Psi_d = \int_A \vec{D}\, d\vec{A}$$

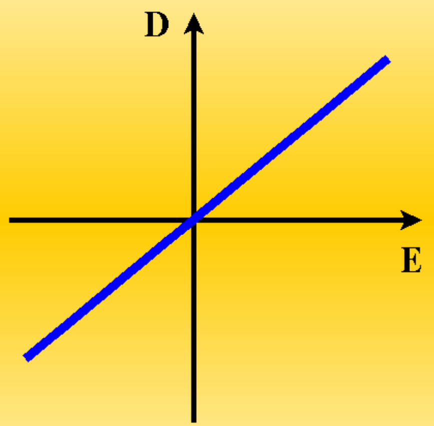
das elektrostatische Feld zweier ungleichnamig geladener Kugelelektroden



daraus ergibt sich als Grundeigenschaft des elektrostatischen Feldes die Beobachtungstatsache:

$$\oint \vec{D}\, d\vec{A} = Q_{umfaßt}$$

- der Zusammenhang zwischen elektrischer Flußdichte und Feldstärke



für elektrisch lineare Werkstoffe gilt:

$$\vec{D} = \varepsilon \vec{E}$$

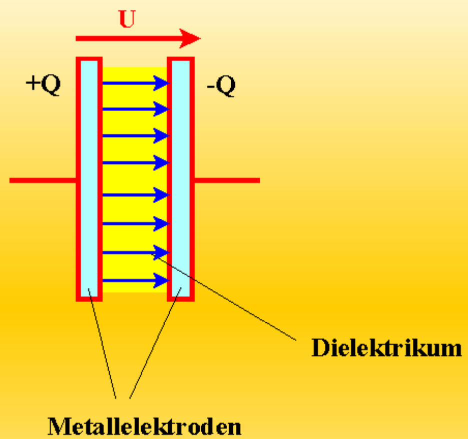
im Vakuum gilt die absolute Permittivität

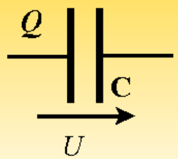
$$\varepsilon_0 = 8{,}86 \cdot 10^{-12}\ \frac{\mathrm{As}}{\mathrm{Vm}}$$

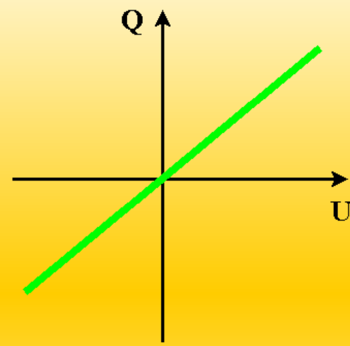
allgemein ist mit der relativen Permittivität:

$$\varepsilon = \varepsilon_r\, \varepsilon_0$$

-Kapazität und Kondensator







$$Q = C\, U$$

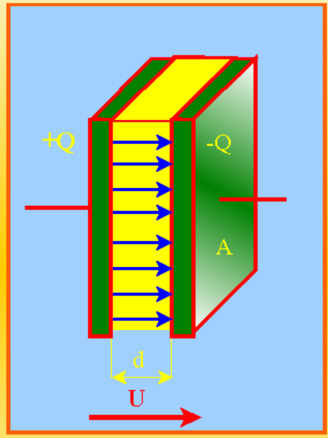
die Proportionalitätskonstante

$$C = \frac{Q}{U}$$

wird als Kapazität bezeichnet.

$$[C] = \frac{[Q]}{[U]} = 1\frac{\mathrm{As}}{\mathrm{V}} = 1\mathrm{F(arad)}$$

Bemessungsgleichung der Kapazität



im homogenen Feld gilt:

$$Q = \Psi_d = D\,A$$

$$U = E\,d$$

damit wird die Kapazität

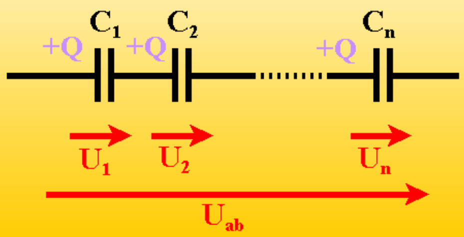
$$C = \frac{Q}{U} = \frac{D\,A}{E\,d} = \frac{\varepsilon E\,A}{E\,d}$$

$$C = \frac{\varepsilon A}{d}$$

ε - Permittivität

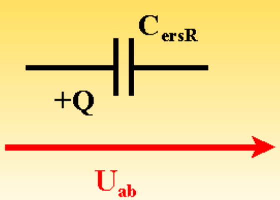
- Zusammenschaltung von Kondensatoren

Reihenschaltung von Kondensatoren



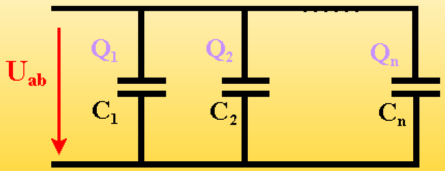
$$Q_1 = Q_2 = \cdots = Q_n = Q$$

$$U_{ab} = \sum_{i=1}^{n} U_i = \sum_{i=1}^{n} \frac{Q}{C_i} = Q\sum_{i=1}^{n} \frac{1}{C_i} = \frac{Q}{C_{ersR}}$$



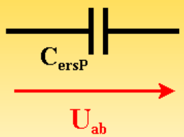
$$\frac{1}{C_{ersR}} = \sum_{i=1}^{n} \frac{1}{C_i}$$

Parallelschaltung von Kondensatoren



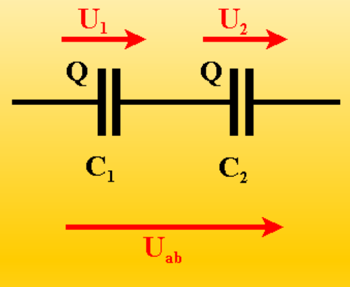
$$U_1 = U_2 = \cdots = U_n = U_{ab}$$

$$Q = \sum_{i=1}^{n} Q_i = \sum_{i=1}^{n} U_{ab} C_i = U_{ab} \sum_{i=1}^{n} C_i = U_{ab} C_{ersP}$$



$$C_{ersP} = \sum_{i=1}^{n} C_i$$

Der kapazitive Spannungsteiler



$$\frac{1}{C_{ersR}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_2 + C_1}{C_1\,C_2}$$

$$C_{ersR} = \frac{C_2\,C_1}{C_1 + C_2}$$

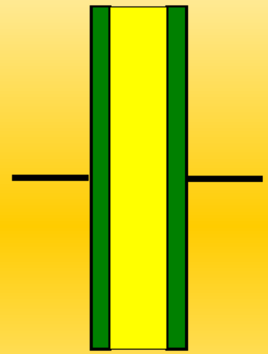
$$Q = C_1 U_1 = C_2 U_2 = C_{ersR}\,U_{ab}$$

$$\frac{U_1}{U_{ab}} = \frac{C_{ersR}}{C_1} = \frac{C_2}{C_1 + C_2}$$

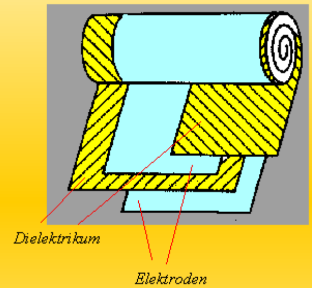
$$\frac{U_1}{U_2} = \frac{C_2}{C_1}$$

- technische Kondensatoren

Kondensatoren mit fester Kapazität

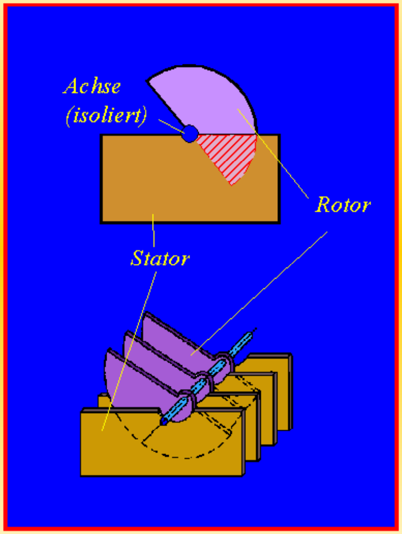


Plattenkondensator

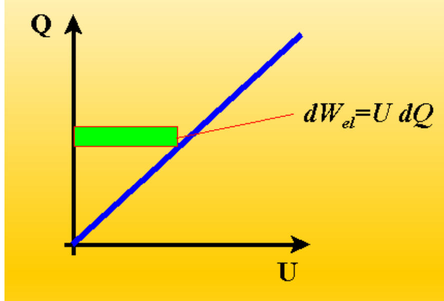


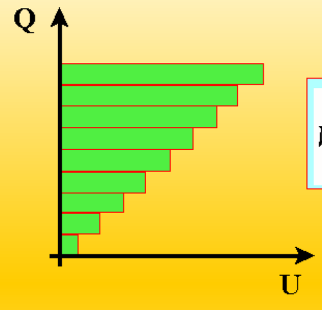
Folienkondensator

Kondensatoren mit variabler Kapazität



Energie im elektrostatischen Feld





$$W_{el} = \int_0^Q U(Q)\,dQ$$

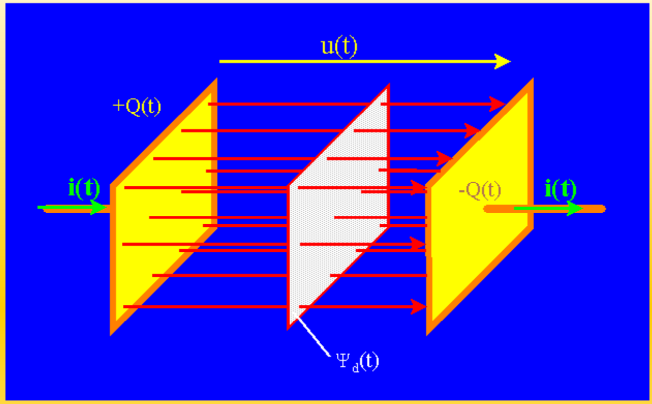
im linearen Fall ergibt sich

$$Q = C\,U$$

$$dQ = C\,dU$$

$$W_{el} = \int_0^U C\,U\,dU = C\frac{U^2}{2} = \frac{QU}{2} = \frac{Q^2}{2C}$$

- der Verschiebungsstrom



Leitungsstrom

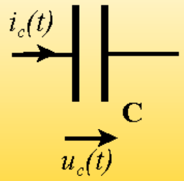
$$i_L = \frac{dQ(t)}{dt}$$

im Nichtleiter gilt

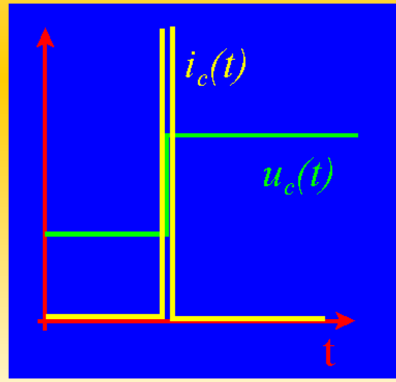
$$Q(t) = C\,u(t)$$

$$i_V = i_L = \frac{dQ(t)}{dt} = C\frac{du(t)}{dt}$$

Zustandsänderungen an Kondensatoren



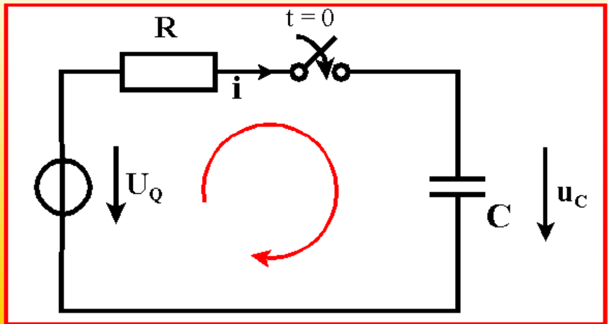
$$i_C(t) = C\frac{du_C}{dt}$$



das Schaltgesetz:

$$u_c(t-0) = u_c(t+0)$$

die Aufladung von Kondensatoren



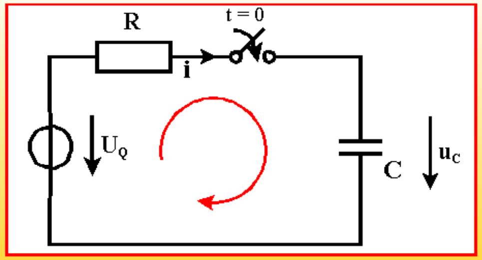
Maschensatz:

$$U_Q = R\,i + u_C$$

$$i = C\frac{du_C}{dt}$$

lineare Differentialgleichung 1. Ordnung mit konstanten Koeffizienten

$$U_Q = RC\frac{du_C}{dt} + u_C$$



$$U_Q = RC\frac{du_C}{dt} + u_C$$

Abkürzung: $RC = \tau$

Lösungsmethode: - Trennung der Variablen

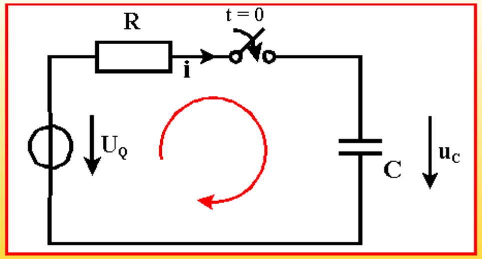
$$U_Q = \tau\frac{du_C}{dt} + u_C$$

$$U_Q\,dt = \tau\,du_C + u_C\,dt$$

$$-\tau\,du_C = u_C\,dt - U_Q\,dt$$

$$\frac{du_C}{u_C - U_Q} = -\frac{dt}{\tau}$$

$$\int\frac{du_C}{u_C - U_Q} = -\int\frac{dt}{\tau}$$



$$\int \frac{du_C}{u_C - U_Q} = -\int \frac{dt}{\tau}$$

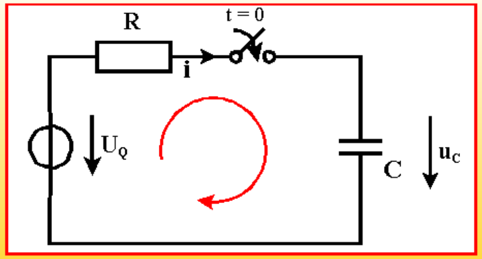
$$\ln\left(u_C - U_Q\right) = -\frac{t}{\tau} + \ln K$$

$$\ln\frac{\left(u_C - U_Q\right)}{K} = -\frac{t}{\tau}$$

$$\frac{u_C - U_Q}{K} = e^{-\frac{t}{\tau}}$$

die allgemeine Lösung:

$$u_C = U_Q + K e^{-\frac{t}{\tau}}$$



$$u_C = U_Q + K e^{-\frac{t}{\tau}}$$

$$u_C = U_Q(1 - e^{-\frac{t}{\tau}})$$

Anfangsbedingung:

$$u_C(0-0) = u_C(0+0) = 0 = U_Q + K e^{-\frac{0}{\tau}}$$

$$K = -U_Q$$

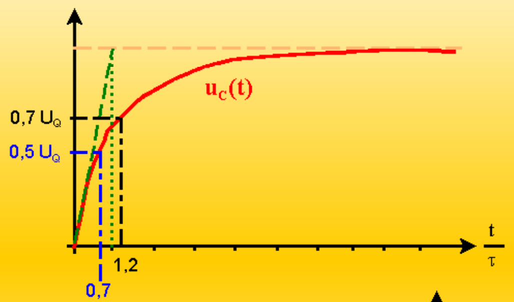
und für den Strom

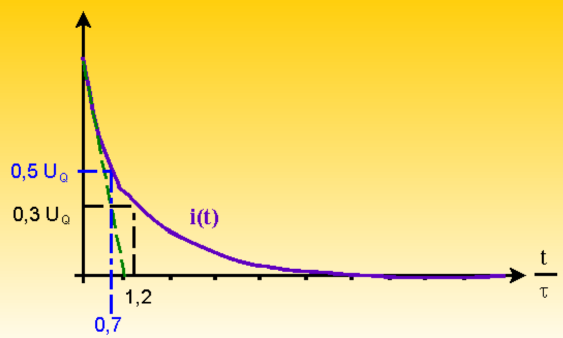
$$i = C\frac{du_C}{dt} = -U_Q C e^{-\frac{t}{\tau}}(-\frac{1}{\tau}) = \frac{U_Q}{R} e^{-\frac{t}{\tau}}$$

Kurvendiskussion:

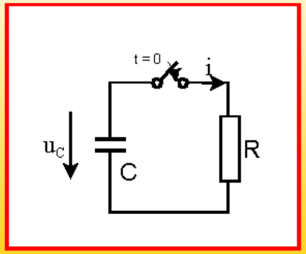
$$u_C = U_Q(1 - e^{-\frac{t}{\tau}})$$

$$i = \frac{U_Q}{R} e^{-\frac{t}{\tau}}$$





- die Entladung von Kondensatoren



$$u_C = U_Q + K e^{-\frac{t}{\tau}}$$

$$u_C = K e^{-\frac{t}{\tau}}$$

Anfangsbedingung:

$$u_C(0-0) = u_C(0+0) = U_0 = K e^{-\frac{0}{\tau}}$$

$$u_C = U_0 e^{-\frac{t}{\tau}}$$

$$K = U_0$$

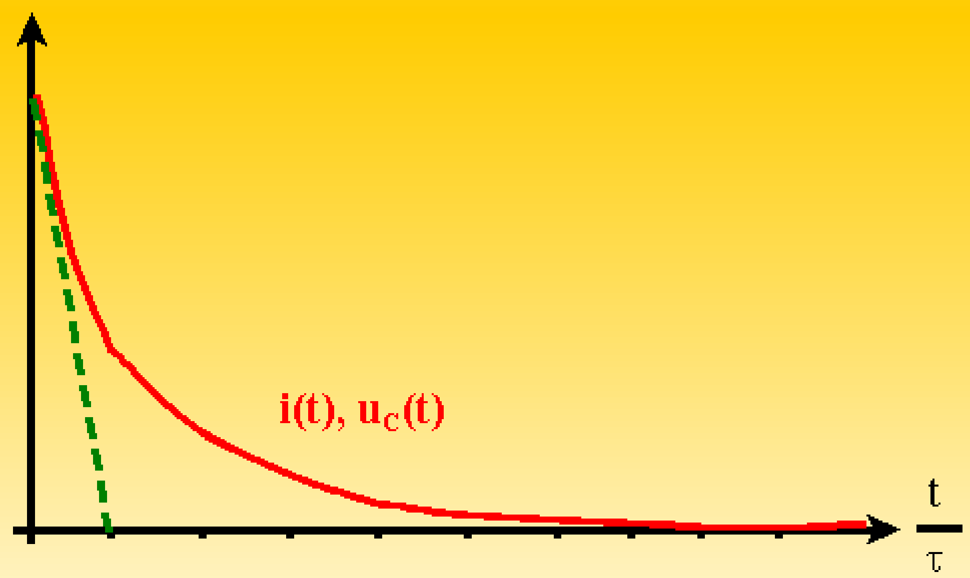
und für den Strom

$$i = -C\frac{du_C}{dt} = -U_0 C e^{-\frac{t}{\tau}}(-\frac{1}{\tau}) = \frac{U_0}{R} e^{-\frac{t}{\tau}}$$

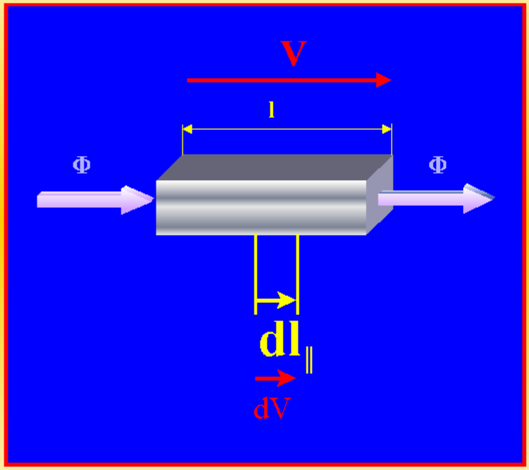
Kurvendiskussion:

$$\boxed{u_C = U_0\, e^{-\frac{t}{\tau}}}$$

$$\boxed{i = \frac{U_Q}{R}\, e^{-\frac{t}{\tau}}}$$



- die magnetische Feldstärke



$$dV = H\,dl_{\parallel}$$

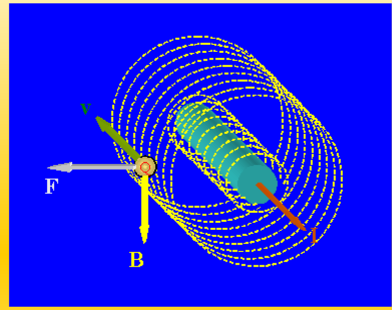
die magnetische Feldstärke

$$V = \int_l \vec{H}\,d\vec{l}$$

$$[H] = \frac{[V]}{[l]} = \frac{\mathrm{A}}{\mathrm{m}}$$

1.7.2 Spule, Induktivität und Gegeninduktivität

- das Magnetfeld



die Lorentzkraft

$$|\vec{F}| \sim Q$$

$$|\vec{F}| \sim |\vec{v}|$$

$$|\vec{F}| \sim \sin\alpha_{(Feilspäne,\vec{v})}$$

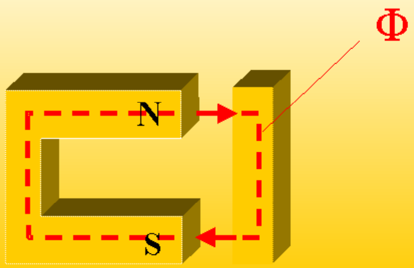
$$|\vec{F}| \sim Q|\vec{v}|\sin\alpha_{(Feilspäne,\vec{v})}$$

$$\vec{F} = Q(\vec{v}\times\vec{B})$$

die Magnetflußdichte

$$[B] = \frac{[F]}{[Q][v]} = 1\frac{N}{As\frac{m}{s}} = 1\frac{\frac{Nm}{m}}{Am} = 1\frac{VAs}{Am^2} = 1\frac{Vs}{m^2} = 1\mathrm{T}$$

- Magnetfluß und Magnetflußdichte



$$\Phi = \int_{A_\perp} B\,dA_\perp$$

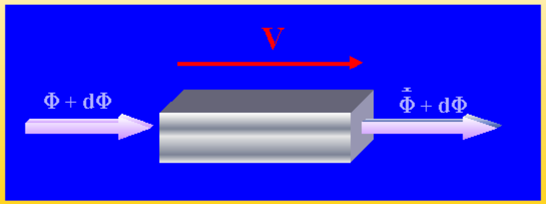
$$\Phi = \int_A \vec{B}\,d\vec{A}$$

$$[\Phi] = [B][A] = 1\frac{Vs}{m^2}m^2 = 1Vs = 1\mathrm{Wb}$$

magnetischer Knotensatz

$$\sum_i \Phi_{i\,vorzeichen} = 0$$

- die magnetische Spannung



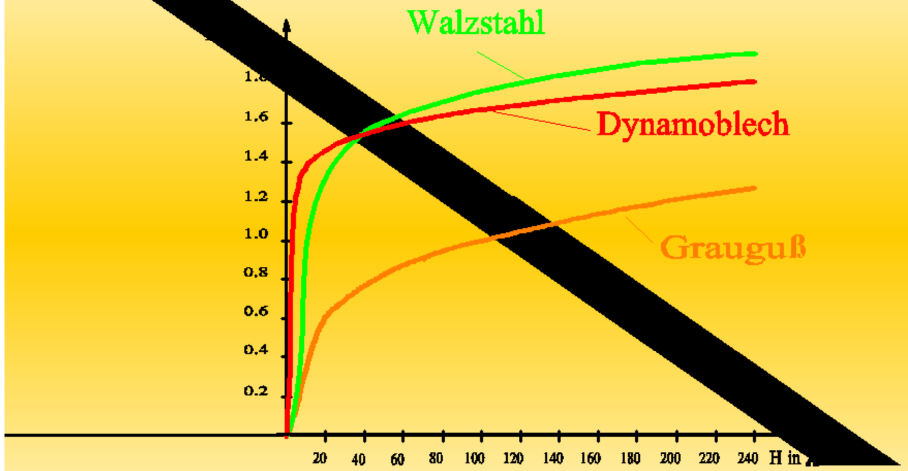
$$dW_{mag} \sim d\Phi$$

$$dW_{mag} = V\,d\Phi$$

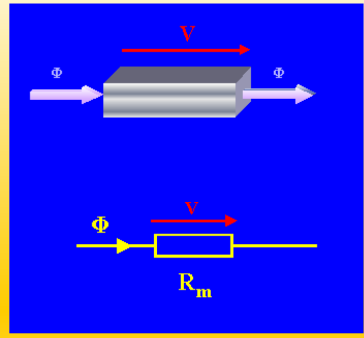
die magnetische Spannung

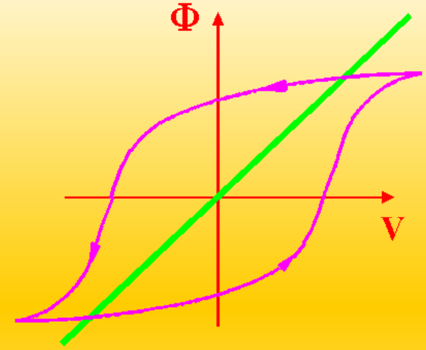
$$[V] = \frac{[W_{mag}]}{[\Phi]} = 1\frac{\mathrm{Ws}}{\mathrm{Vs}} = 1\frac{\mathrm{VAs}}{\mathrm{Vs}} = 1\mathrm{A}$$

- [redacted]ie Kommutierungskennlinie einiger Werkstoffe



- der magnetische Widerstand

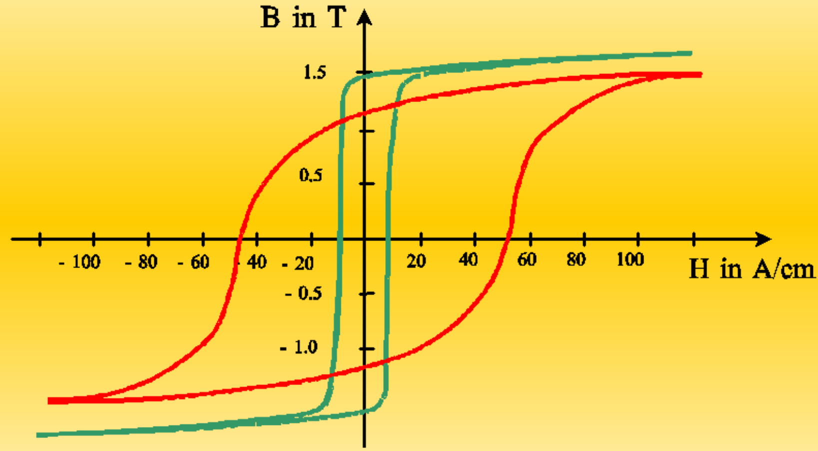




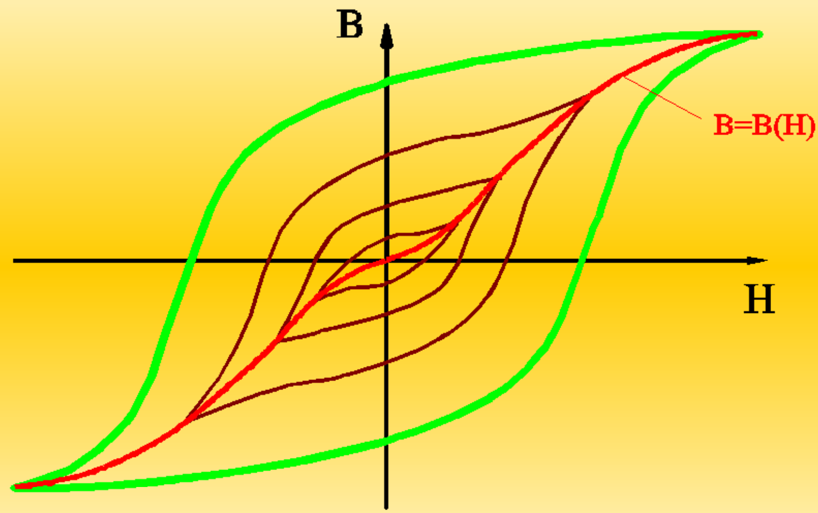
$$\boxed{V = R_m \Phi}$$

$$\boxed{R_m = \frac{V}{\Phi}}$$

- hart- und weichmagnetische Werkstoffe



- die Kommutierungskennlinie



- die Bemessungsgleichung des magnetischen Widerstands

$$R_m = \frac{V}{\Phi}$$

im homogenen Feld gilt

$$V = H\,l \qquad \text{und} \qquad \Phi = B\,A$$

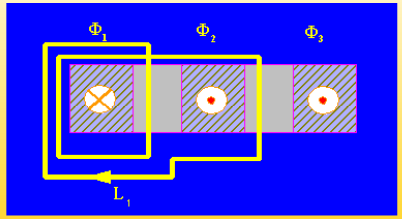
damit wird

$$R_m = \frac{H\,l}{B\,A} = \frac{H\,l}{\mu H\,A}$$

$$R_m = \frac{l}{\mu\;A}$$

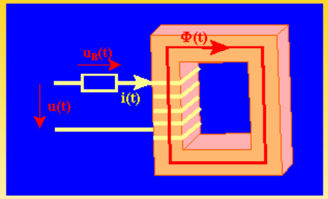
- Das Faradaysche Induktionsgesetz

Der verkettete Fluß



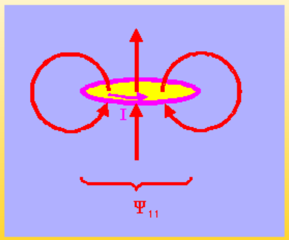
$$\Psi = \sum_i \Phi_{\,i\ \text{umfaßt vorzeichen}} \qquad \Psi = 2\Phi_1 - \Phi_2$$

Das Faradaysche Induktionsgesetz



$$u_{ind} = +\frac{d\Psi}{dt}$$

- Selbstinduktion und Induktivität



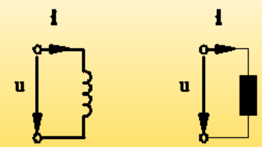
$$I \to B \to \Phi \to \Psi$$

$$\Psi = L\,I$$

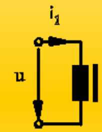
$$L = \frac{\Psi}{I}$$ **Induktivität**

$$[L] = \frac{[\Psi]}{[I]} = 1\frac{\text{Vs}}{\text{A}} = 1\text{H}$$

Schaltzeichen für Induktivitäten



Spule mit Eisenkreis

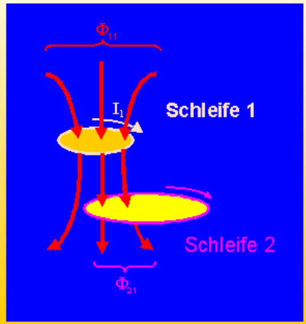


Strom - Spannungsgleichung :

$$u = +\frac{d\Psi}{dt}$$

$$u = +L\frac{d\,i}{dt}$$

- **Gegeninduktion und Gegeninduktivität**

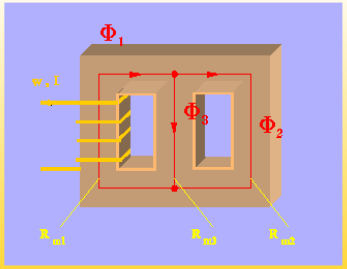


$$\boxed{I_1 \rightarrow B \rightarrow \Phi_{11} \rightarrow \Phi_{21} \rightarrow \Psi_{21}}$$

Gegeninduktivität $\boxed{\Psi_{21} = M I_1}$

$$\boxed{M = \frac{\Psi_{21}}{I_1} = \frac{\Psi_{12}}{I_2}}$$

$$\boxed{[M] = \frac{[\Psi]}{[I]} = 1\frac{\mathrm{Vs}}{\mathrm{A}} = 1\mathrm{H}}$$

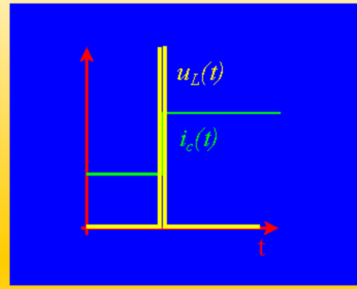


$$\boxed{L = \frac{\Psi}{I}}$$

$$\boxed{L = \frac{w\Phi_1}{I}}$$

$$\boxed{L = \frac{w\Phi_1}{I} = \frac{w}{I}\frac{\Theta}{R_{mers}} = \frac{w}{I}\frac{wI}{R_{mers}} = \frac{w^2}{R_{mers}}}$$

Zustandsänderungen an Induktivitäten

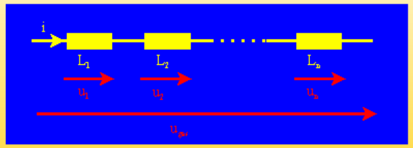


das Schaltgesetz:

$$\boxed{i_L(t-0) = i_L(t+0)}$$

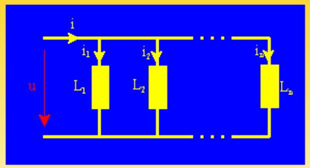
- Induktivität in Schaltungen

Reihenschaltung von Induktivitäten



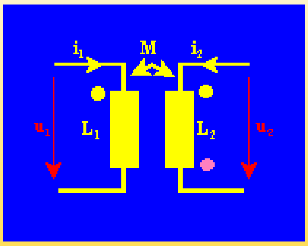
$$\boxed{L_{ersR} = \sum_{i=1}^{n} L_i}$$

Parallelschaltung von Induktivitäten



$$\boxed{\frac{1}{L_{ersP}} = \sum_{i=1}^{n} \frac{1}{L_i}}$$

Schaltzeichen für Gegeninduktivitäten



Strom - Spannungsgleichung :

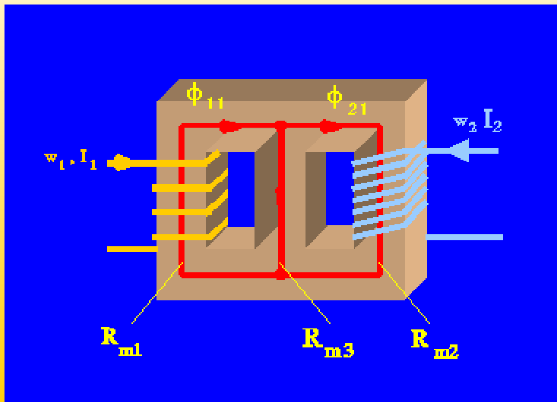
$$u = +\frac{d\Psi}{dt}$$

$$u_j = +\frac{d(Mi_k)}{dt}$$

$$u_j = \pm M\frac{d\,i_k}{dt}$$

$$u_1 = +L_1\frac{d\,i_1}{dt} \pm M\frac{d\,i_2}{dt}$$

die Gegeninduktivitätsbemessungsgleichung



$$M = k\sqrt{L_1 L_2}$$

$$k = \sqrt{k_{12}k_{21}}$$

$$k_{21} = \frac{\Phi_{21}}{\Phi_{11}} \qquad k_{12} = \frac{\Phi_{12}}{\Phi_{22}}$$