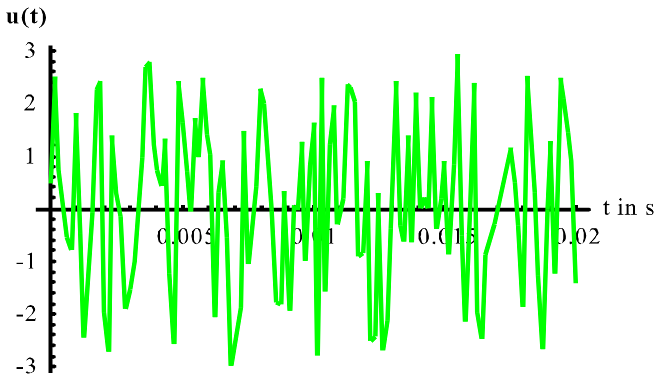
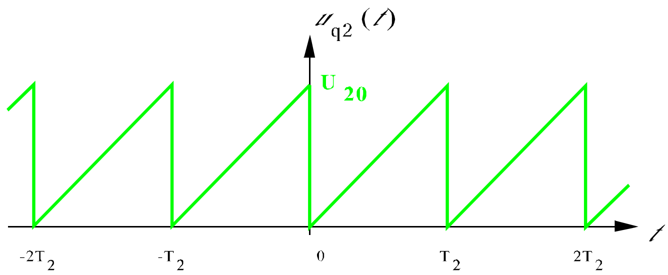
2. Die Übertragung von Informationen durch elektrische Wechselgrößen

2.1 Die Arten von Wechselgrößen

- Zufallsprozesse



- determinierte Signale (nichtsinusförmig und periodisch)

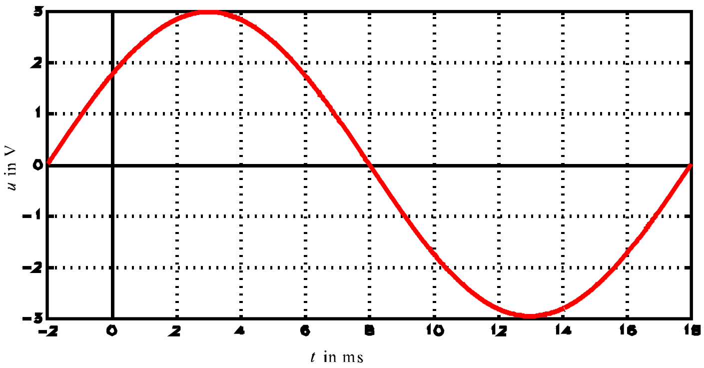


$$x(t) = x(t + nT) \quad n\text{ – ganz} \quad T\text{ - Periodendauer}$$

$$f = \frac{1}{T} \quad f\text{ – Frequenz}$$

$$[f] = \frac{1}{[T]} = 1\mathrm{s}^{-1} = 1\mathrm{Hz}$$

- determinierte Signale (sinusförmig)



$$u(t) = \hat{U} \sin(\omega t + \varphi_u)$$

$$\omega = 2\pi \frac{1}{T} = 2\pi f \quad \text{in s}^{-1}$$

$$f = \frac{1}{T} \quad \text{in Hz}$$

- **Gleichrichtwert**

$$\overline{X} = \frac{1}{T}\int_{t_0}^{t_0+T} |x(t)|\,dt$$

für sinusförmige Wechselgrößen:

$$\overline{X} = \frac{2}{\pi}\hat{X}$$

- **Effektivwert**

$$p_{el} = u\,i = \frac{u^2}{R} = i^2 R$$

$$\overline{P} = \frac{1}{T}\int_{t_0}^{t_0+T} \frac{u^2(t)}{R}\,dt = \frac{U^2}{R}$$

$$U = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} u^2(t)\,dt}$$

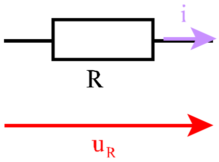
$$X = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} x^2(t)\,dt}$$

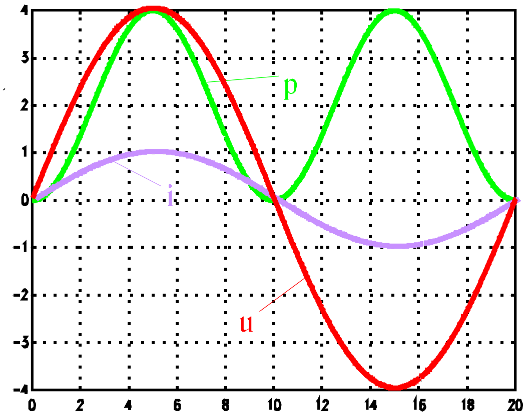
für sinusförmige Wechselgrößen:

$$X = \frac{\hat{X}}{\sqrt{2}}$$

3.3 Die Reaktion der Bauelemente R, L, C auf sinusförmige Erregung

1. Widerstand





$$\boxed{i = \hat{i}\sin(\omega t + \varphi_i)}$$

$$\boxed{u_R = Ri}$$

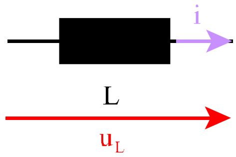
$$\boxed{u_R = R\hat{i}\sin(\omega t + \varphi_i) = \hat{u}_R\sin(\omega t + \varphi_{uR})}$$

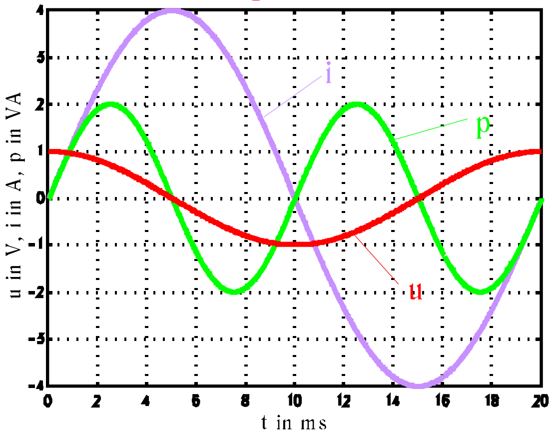
$$\boxed{\hat{u}_R = R\hat{i}}$$

$$\boxed{\varphi_{uR} = \varphi_i}$$

$$\boxed{\Delta\varphi = \varphi_{uR} - \varphi_i = 0}$$

2. Induktivität





$$i = \hat{I}\sin(\omega t + \varphi_i)$$

$$u_L = L\frac{di}{dt}$$

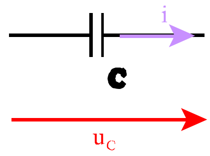
$$u_L = \omega L\hat{I}\cos(\omega t + \varphi_i) = \omega L\hat{I}\sin(\omega t + \varphi_i + \frac{\pi}{2}) = \hat{U}_R\sin(\omega t + \varphi_{uR})$$

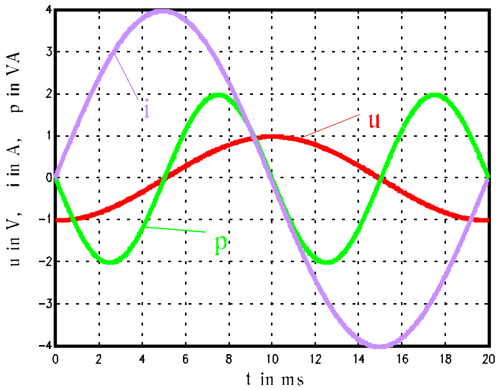
$$\hat{U}_L = \omega L\hat{I}$$

$$\varphi_{uR} = \varphi_i + \frac{\pi}{2}$$

$$\Delta\varphi = \varphi_{uR} - \varphi_i = \frac{\pi}{2}$$

3. Kapazität





$$i = \hat{I} \sin(\omega t + \varphi_i)$$

$$u_C = \frac{1}{C}\int i\, dt$$

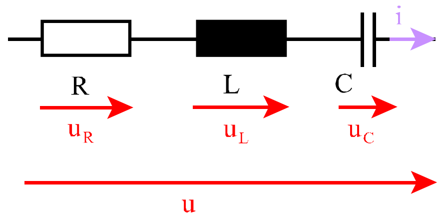
$$u_C = -\frac{1}{\omega C}\hat{I}\cos(\omega t + \varphi_i) = \frac{1}{\omega C}\hat{I}\sin(\omega t + \varphi_i - \frac{\pi}{2}) = \hat{U}_C \sin(\omega t + \varphi_{uC})$$

$$\hat{U}_C = \frac{1}{\omega C}\hat{I}$$

$$\varphi_{uC} = \varphi_i - \frac{\pi}{2}$$

$$\Delta\varphi = \varphi_{uC} - \varphi_i = -\frac{\pi}{2}$$

4. Reihenschaltung von R, L und C



$$\boxed{u = \hat{U}\sin(\omega t + \varphi_u)}$$

$$\boxed{u_R = R\,i} \qquad \boxed{u_L = L\frac{di}{dt}} \qquad \boxed{u_C = \frac{1}{C}\int i\,dt}$$

$$\boxed{i\,R + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = \hat{U}\sin(\omega t + \varphi_u)}$$

bekannte Merkmale der Lösung :

1. In linearen Netzen sind bei sinusförmiger Erregung alle stationären Vorgänge gleichfalls sinusförmig.
2. In linearen Netzwerken bleibt die Frequenz erhalten.

noch benötigte Informationen :

1. Amplitude der Lösung
2. Startphase der Lösung

Ansatz : $\boxed{i = \hat{I}\sin(\omega t + \varphi_i)}$

Ergebnis:

$$i(t) = \frac{\hat{U}}{\sqrt{R^2 + \left[\omega L - \frac{1}{\omega C}\right]^2}} \sin(\omega t + \varphi_i) = \frac{\hat{U}}{Z} \sin(\omega t + \varphi_u - \varphi_Z)$$

mit

$$Z = \sqrt{R^2 + \left[\omega L - \frac{1}{\omega C}\right]^2}$$

Scheinwiderstand (Impedanz)

$$X = \left[\omega L - \frac{1}{\omega C}\right]$$

Blindwiderstand (Reaktanz)

$$X_L = \omega L$$

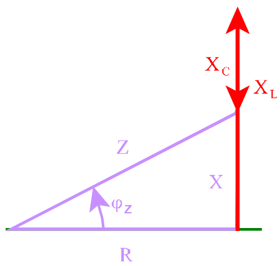
induktiver Blindwiderstand

$$X_C = -\frac{1}{\omega C}$$

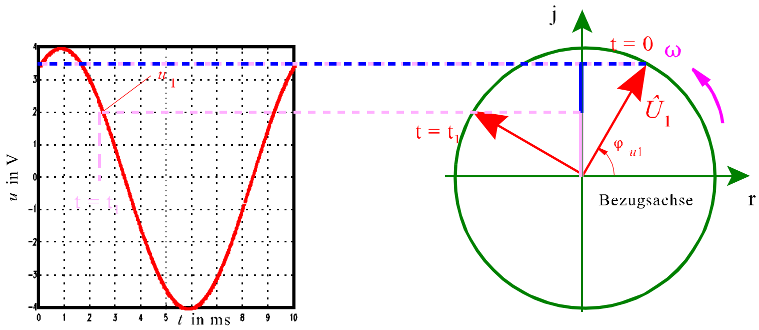
kapazitiver Blindwiderstand

$$Z = \sqrt{R^2 + X^2}$$

$$\varphi_Z = \arctan\left[\frac{\omega L - \frac{1}{\omega C}}{R}\right]$$



2.4 Darstellung sinusförmiger Wechselgrößen in der Gaußschen Zahlenebene



$$u_1 = \hat{U}_1 \sin(\omega t + \varphi_{u1})$$

$$\sqrt{-1} = j$$

rotierender Zeiger oder komplexer Momentanwert

$$\underline{u}_1 = Re\{\underline{u}_1\} + j\,\mathrm{Im}\{\underline{u}_1\} = \hat{\underline{U}}_1\left(\cos(\omega t + \varphi_{u1}) + j\sin(\omega t + \varphi_{u1})\right) = \hat{\underline{U}}_1 e^{j(\omega t + \varphi_{u1})}$$

$$u_1 = \hat{U}_1 \sin(\omega t + \varphi_{u1}) = \mathrm{Im}\{\underline{u}_1\}$$

ruhender Zeiger oder komplexe Amplitude

$$\hat{\underline{U}}_1 = \frac{\hat{U}_1 e^{j(\omega t + \varphi_{u1})}}{e^{j\omega t}} = \hat{U}_1 e^{j\varphi_{u1}}$$

komplexer Effektivwert

$$\underline{U}_1 = \frac{\hat{\underline{U}}_1}{\sqrt{2}} = \frac{\hat{U}_1}{\sqrt{2}} e^{j\varphi_{u1}} = U_1 e^{j\varphi_{u1}}$$

- die Anwendung der komplexen Rechnung zur Berechnung von Wechselstromschaltungen

Rechenregeln im

	Zeitbereich	Bildbereich (komplexe Ebene)
Knotensatz	$\sum i_{n\,vorz} = 0$	$\sum \mathrm{Im}\left\{\underline{i}_{n\,vorz}\right\} = 0 \rightarrow \sum \underline{i}_{n\,vorz} = 0 \rightarrow \sum \underline{\hat{I}}_{n\,vorz} = 0$
Maschensatz	$\sum u_{n\,vorz} = 0$	$\sum \mathrm{Im}\left\{\underline{u}_{n\,vorz}\right\} = 0 \rightarrow \sum \underline{u}_{n\,vorz} = 0 \rightarrow \sum \underline{\hat{U}}_{n\,vorz} = 0$
Widerstand	$u_R = R i_R$	$\mathrm{Im}\left\{\underline{u}_R\right\} = R\,\mathrm{Im}\left\{\underline{i}_R\right\} \rightarrow \underline{u}_R = R\,\underline{i}_R \rightarrow \underline{\hat{U}}_R = R\,\underline{\hat{I}}_R$
Induktivität	$u_L = L\frac{di_L}{dt}$	$\underline{u}_L = L\frac{d\underline{i}_L}{dt} = L\frac{d\hat{I}e^{j(w\,t+j_i)}}{dt}$
		$\underline{u}_L = jw\,L\,\hat{I}e^{j(w\,t+j_i)} = jw\,L\,\underline{i}$
		$\underline{u}_L = jw\,L\underline{i} \rightarrow \underline{\hat{U}}_L = jw\,L\,\underline{\hat{I}}$
Kapazität	$u_C = \frac{1}{C}\int i_C\,dt$	$\underline{u}_C = \frac{1}{C}\int \underline{i}_C\,dt = \frac{1}{C}\int \hat{I}e^{j(w\,t+j_i)}\,dt$
		$\underline{u}_C = \frac{1}{jw\,C}\int \hat{I}e^{j(w\,t+j_i)}\,d(jw\,t) = \frac{1}{jw\,C}\hat{I}e^{j(w\,t+j_i)}$
		$\underline{u}_C = \frac{1}{jw\,C}\underline{i} \rightarrow \underline{\hat{U}}_C = \frac{1}{jw\,C}\underline{\hat{I}}$

Zusammenfassung der Rechenregeln im Komplexen:

Knotensatz

$$\sum \hat{\underline{I}}_{n\,vorz} = 0$$

Maschensatz

$$\sum \hat{\underline{U}}_{n\,vorz} = 0$$

komplexer Widerstand

$$\hat{\underline{U}} = \underline{Z}\ \hat{\underline{I}}$$

mit

$$\underline{Z}_R = R$$

$$\underline{Z}_L = j\omega\ L$$

$$\underline{Z}_C = \frac{1}{j\omega\ C}$$

komplexer Leitwert

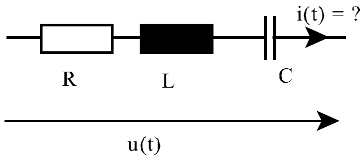
$$\hat{\underline{I}} = \underline{Y}\,\hat{\underline{U}}$$

mit

$$\underline{\boldsymbol{Y}} = \frac{\boldsymbol{1}}{\underline{\boldsymbol{Z}}}$$

Anwendungsbeispiel:

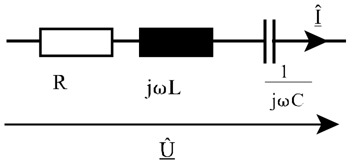
Aufgabe:



$$u(t) = \hat{U}\sin(\omega t + \varphi_U)$$

Lösung:

1. Transformation ins Komplexe



$$\underline{\hat{U}} = \hat{U} e^{j\varphi_U}$$

2. Ermittlung der Lösung im Komplexen

$$\underline{\hat{I}} = \frac{\underline{\hat{U}}}{\underline{Z}_{ges}} = \frac{\underline{\hat{U}}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{\hat{U} e^{j\varphi_U}}{R + j\omega L + \frac{1}{j\omega C}}$$

3. Berechnung der Exponentialform der Lösung

$$\underline{\hat{I}} = \frac{\hat{U} e^{j\varphi_U}}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{\hat{U} e^{j\varphi_U}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\, e^{j\varphi_Z}}$$

$$\varphi_Z = \arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

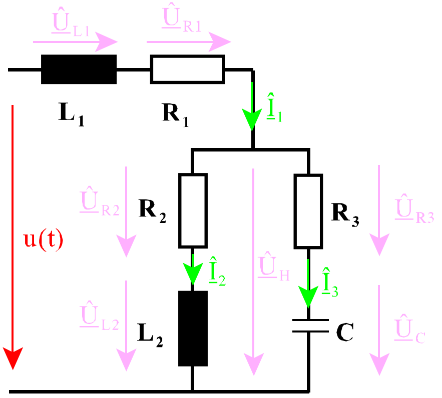
4. Rücktransformation in den Zeitbereich

$$i(t) = \frac{\hat{U}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin(\omega t + \varphi_U - \varphi_Z)$$

- Grafische Methoden der Wechselstromtechnik

Das topologische Zeigerdiagramm

Beispiel



$u(t) = \hat{U}\sin(\omega t + \varphi_u)$ $\quad \underline{\hat{U}} = \hat{U}\,e^{j\varphi u}$

Ansatz: $i_3(t) = \hat{I}_3 \sin\omega t$ $\quad \underline{\hat{I}}_3 = \hat{I}_3$

$\underline{\hat{U}}_{R3} = R_3\, \underline{\hat{I}}_3$

$\underline{\hat{U}}_C = 1/j\omega C\, \underline{\hat{I}}_3$

$\underline{\hat{U}}_H = \underline{\hat{U}}_{R3} + \underline{\hat{U}}_C$

$\varphi_{I2} = \varphi_{UH} - \arctan(\omega L_2/R_2)$

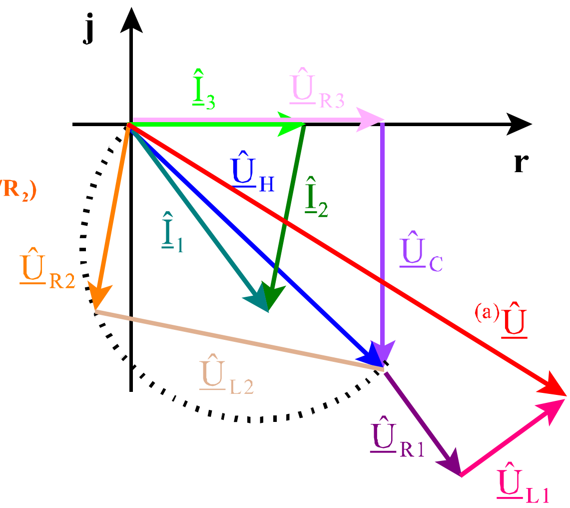
$\underline{\hat{I}}_2 = \underline{\hat{U}}_{R2}/R_2$

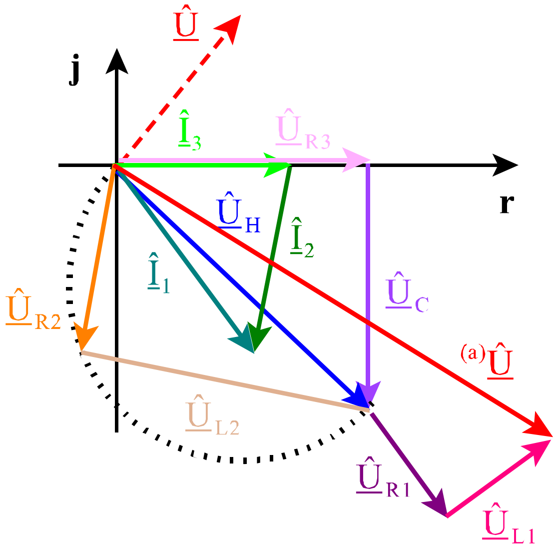
$\underline{\hat{I}}_1 = \underline{\hat{I}}_2 + \underline{\hat{I}}_3$

$\underline{\hat{U}}_{R1} = R_1\, \underline{\hat{I}}_1$

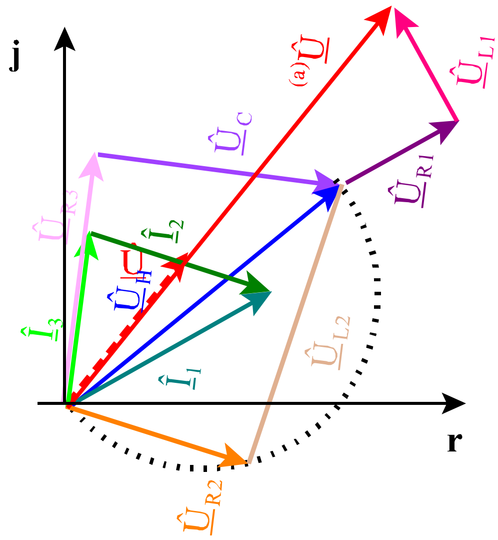
$\underline{\hat{U}}_{L1} = j\omega L_1\, \underline{\hat{I}}_1$

$^{(a)}\underline{\hat{U}} = \underline{\hat{U}}_H + \underline{\hat{U}}_{R1} + \underline{\hat{U}}_{L1}$



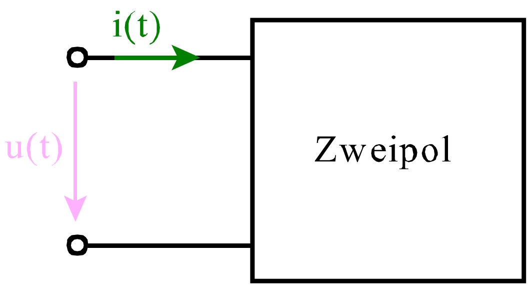


$$\frac{\underline{\hat{X}}}{{}^{(a)}\underline{\hat{X}}} = \frac{\underline{\hat{U}}}{{}^{(a)}\underline{\hat{U}}} \qquad \underline{\hat{X}} = \frac{\underline{\hat{U}}}{{}^{(a)}\underline{\hat{U}}}\,{}^{(a)}\underline{\hat{X}} = \left[\frac{\underline{\hat{U}}}{{}^{(a)}\underline{\hat{U}}}\right] e^{j\left(\varphi_U - {}^{(a)}\varphi_U\right)}\,{}^{(a)}\underline{\hat{X}}$$



- Die Leistung in der Wechselstromtechnik

Die Leistung im Zeitbereich



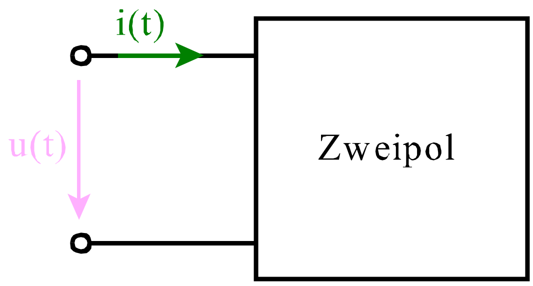
$$u(t) = \hat{U}\sin\omega t$$

$$i(t) = \hat{I}\sin\left(\omega t - \varphi_{U,I}\right)$$

$$p(t) = u(t)\,i(t) = \hat{U}\sin\omega t\;\hat{I}\sin\left(\omega t - \varphi_{U,I}\right)$$

$$p(t) = \hat{U}\hat{I}\sin\omega t\left(\sin\omega t\cos\varphi_{U,I} - \cos\omega t\sin\varphi_{U,I}\right)$$

$$p(t) = \hat{U}\hat{I}\left(\sin^2\omega t\cos\varphi_{U,I} - \sin\omega t\cos\omega t\sin\varphi_{U,I}\right)$$



$$\boxed{p(t) = \hat{U}\hat{I}\left(\sin^2\omega t \cos\varphi_{U,I} - \sin\omega t \cos\omega t \sin\varphi_{U,I}\right)}$$

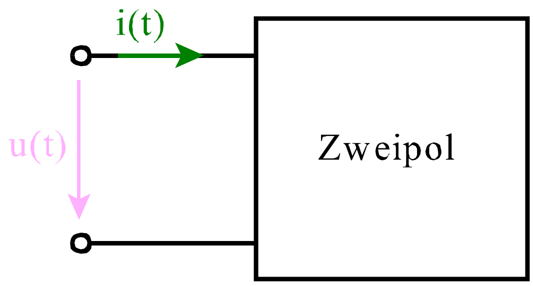
$$\boxed{\sin^2\omega t = \frac{1}{2}\left(1 - \cos 2\omega t\right)}$$

$$\boxed{\sin\omega t \cos\omega t = \frac{1}{2}\sin 2\omega t}$$

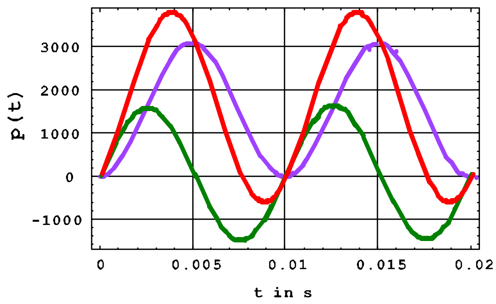
$$\boxed{p(t) = \frac{\hat{U}\hat{I}}{2}\left(\cos\varphi_{U,I}\left(1 - \cos 2\omega t\right) - \sin\varphi_{U,I}\sin 2\omega t\right)}$$

$$\boxed{p(t) = \frac{\hat{U}\hat{I}}{\sqrt{2}\sqrt{2}}\left(\cos\varphi_{U,I}\left(1 - \cos 2\omega t\right) - \sin\varphi_{U,I}\sin 2\omega t\right)}$$

$$\boxed{p(t) = UI\cos\varphi_{U,I}\left(1 - \cos 2\omega t\right) - UI\sin\varphi_{U,I}\sin 2\omega t}$$



$$p(t) = UI\cos\varphi_{U,I}\left(1-\cos 2\omega t\right) - UI\sin\varphi_{U,I}\sin 2\omega t$$



$P = \overline{P} = UI\cos\varphi_{U,I}$ Wirkleistung [W]

$Q = UI\sin\varphi_{U,I}$ Blindleistung [var]

$S = UI$ Scheinleistung [VA]

$\cos\varphi_{U,I}$ Leistungsfaktor

$$\boxed{P = UI\cos\varphi_{U,I}}$$

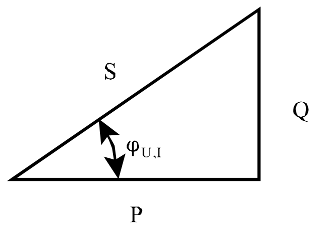
$$\boxed{Q = UI\sin\varphi_{U,I}}$$

$$\boxed{S = UI}$$

$$S = \sqrt{P^2 + Q^2}$$

$$\tan\varphi_{U,I} = \frac{Q}{P}$$

Das Leistungsdreieck



Die Leistung im Bildbereich

Ansatz :

$$\underline{u} = \hat{U} e^{j(\omega t+\varphi_u)} \qquad \underline{i} = \hat{I} e^{j(\omega t+\varphi_i)}$$

$$\underline{p} = \underline{u}\,\underline{i} = \hat{U} e^{j(\omega t+\varphi_u)} \hat{I} e^{j(\omega t+\varphi_i)}$$

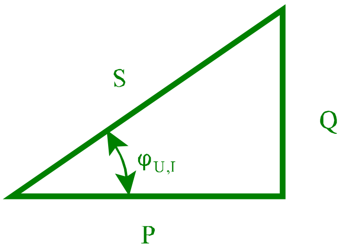
$$\underline{p} = \hat{U} \hat{I} e^{j(2\omega t+\varphi_u+\varphi_i)}$$

wirklicher Leistungsumsatz im Zeitbereich :

$$p(t) = UI\cos\varphi_{U,I}\left(1-\cos 2\omega t\right) - UI\sin\varphi_{U,I}\sin 2\omega t$$

$$\cancel{p(t) = \operatorname{Im}\{\underline{p}\} = \hat{U}\hat{I}\sin\left(2\omega t+\varphi_u+\varphi_i\right)}$$

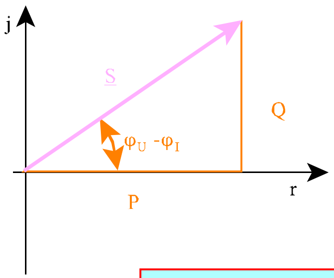
Nachbildung des Leistungsdreiecks im Komplexen:



$$S = UI$$

$$P = UI\cos\varphi_{U,I} = UI\cos(\varphi_U - \varphi_I)$$

$$Q = UI\sin\varphi_{U,I} = UI\sin(\varphi_U - \varphi_I)$$



$$\underline{U} = U\,e^{j\varphi_u}$$

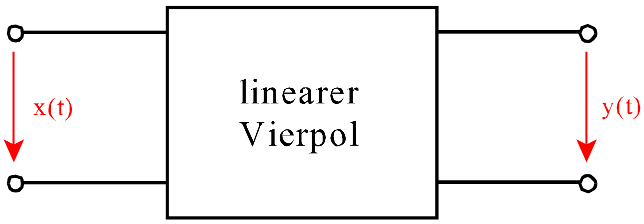
$$\underline{I} = I\,e^{j\varphi_i}$$

$$\underline{S} = \underline{U}\,\underline{I}^* = U\,e^{j\varphi_u} I e^{-j\varphi_i} = UIe^{j(\varphi_u - \varphi_i)}$$

$$P = UI\cos(\varphi_U - \varphi_I) = \mathrm{Re}\{\underline{S}\}$$

$$Q = UI\sin(\varphi_U - \varphi_I) = \mathrm{Im}\{\underline{S}\}$$

2.5 Frequenzkennlinien und Übertragungsverhalten linearer Wechselstromschaltungen



Zeitbereich: $y(t) = f(x(t))$

Bildbereich (komplexe Ebene):

$$\underline{\hat{Y}}(j\omega) = \underline{H}(j\omega)\,\underline{\hat{X}}(j\omega)$$

Übertragungsfunktion: $\underline{H}(j\omega) = \dfrac{\underline{\hat{Y}}(j\omega)}{\underline{\hat{X}}(j\omega)}$

$$\underline{\hat{Y}}(j\omega) = \underline{H}(j\omega)\,\underline{\hat{X}}(j\omega)$$

$$\boxed{\underline{\hat{Y}}(j\omega) = \underline{H}(j\omega)\,\underline{\hat{X}}(j\omega)}$$

$$\boxed{\hat{Y}(\omega)\,e^{j\varphi_y(\omega)} = H(\omega)\,e^{j\varphi_H(\omega)}\,\hat{X}(\omega)\,e^{j\varphi_x(\omega)}}$$

Amplitudenübertragung:

$$\boxed{\hat{Y}(\omega) = H(\omega)\,\hat{X}(\omega)}$$

Amplitudenfrequenzgang des Systems

Phasenübertragung:

$$\boxed{\varphi_y(\omega) = \varphi_H(\omega) + \varphi_x(\omega)}$$

Phasenfrequenzgang des Systems

Amplitudenübertragung:

$$\boxed{\hat{Y}(\omega) = H(\omega)\,\hat{X}(\omega)}$$

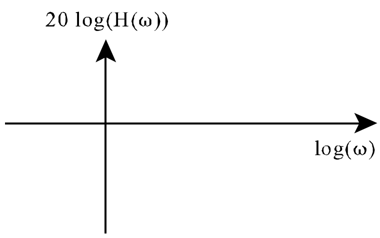
Phasenübertragung:

$$\boxed{\varphi_y(\omega) = \varphi_H(\omega) + \varphi_x(\omega)}$$

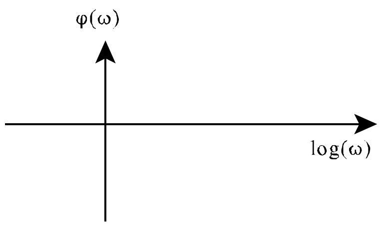
$$\boxed{\log\left(\hat{Y}(\omega)\right) = \log\left(H(\omega)\right) + \log\left(\hat{X}(\omega)\right)}$$

$$\boxed{20\log\left(\hat{Y}(\omega)\right) = 20\log\left(H(\omega)\right) + 20\log\left(\hat{X}(\omega)\right) \quad \text{in dB}}$$

- übliche grafische Darstellung:

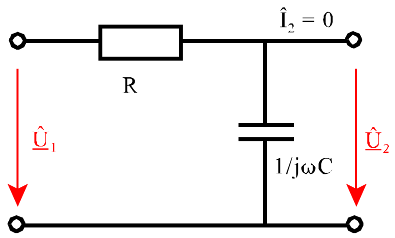


Amplitudenfrequenzgang



Phasenfrequenzgang

Beispiel:



RC - Glied als Tiefpaß

$$\hat{U}_1 = R\hat{I} + \frac{1}{j\omega C}\hat{I}$$

$$\hat{U}_2 = \frac{1}{j\omega C}\hat{I}$$

$$\hat{U}_1 = R\hat{I} + \frac{1}{j\omega C}\hat{I} = \left(R + \frac{1}{j\omega C}\right)\hat{I}$$

$$\hat{U}_2 = \frac{1}{j\omega C}\hat{I} = \frac{1}{j\omega C}\frac{\hat{U}_1}{R + \frac{1}{j\omega C}}$$

$$\underline{H}(j\omega) = \frac{\underline{\hat{U}}_2(j\omega)}{\underline{\hat{U}}_1(j\omega)} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$\underline{H}(j\omega) = \frac{\underline{\hat{U}}_2(j\omega)}{\underline{\hat{U}}_1(j\omega)} = \frac{\dfrac{1}{j\omega C}}{R + \dfrac{1}{j\omega C}}$$

$$\underline{H}(j\omega) = \frac{\underline{\hat{U}}_2(j\omega)}{\underline{\hat{U}}_1(j\omega)} = \frac{1}{1 + j\omega CR}$$

Amplitudenfrequenzgang:

$$H(\omega) = \frac{\hat{U}_2(\omega)}{\hat{U}_1(\omega)} = \frac{1}{\sqrt{1 + (\omega CR)^2}}$$

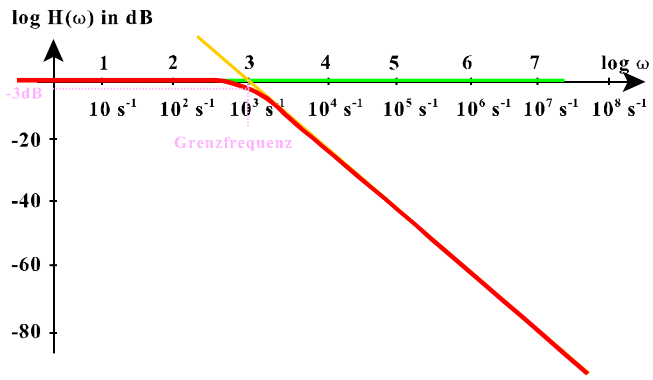
Phasenfrequenzgang:

$$\varphi_H(\omega) = \varphi_{u2}(\omega) - \varphi_{u1}(\omega) = -\arctan \omega CR$$

$$H(\omega) = \frac{1}{\sqrt{1+(\omega\, CR)^2}}$$

$$\frac{\omega}{\omega_O} \ll 1 \qquad 20\log[H(\omega)] \approx 0$$

$$\frac{\omega}{\omega_O} \gg 1 \qquad 20\log[H(\omega)] \approx -20\log\omega + 20\log\omega_O$$



$$\varphi_H(\omega) = -\arctan\left(\frac{\omega}{\omega_O}\right) \qquad \text{mit} \quad \omega_O = \frac{1}{CR}$$