Computation and interpretation of odds ratios in multinomial logistic regression

In the familiar context of 2x2 tables, and from a conceptual point of view, the odds ratio can be computed as follows:

$$OR = \frac{a \div b}{c \div d}$$

where *a, b, c*, and *d* are cell frequencies as shown below, with 2 levels of a predictor variable in rows 1 and 2, and 2 levels of a dependent variable (or outcome variable) in columns 1 and 2. (We are aware of the more commonly used computational formula, OR=(ad)/(bc), but feel that the conceptual formula is more transparent in this context, and therefore more instructive.)

| | Column 1 | Column 2 |
|---|---|---|
| Row 1 | *a* | *b* |
| Row2 | *c* | *d* |

When there is only one predictor variable, the odds ratios produced by multinomial (or polychotomous) logistic regression can be computed in a similar fashion. To see how, let us return to the crosstabulation of Pre-admission Functional Status (as assessed by the attending physician) and Code Status:

| | Code Status | | |
|---|---|---|---|
| Functional Status | Explicit: Resuscitate | Explicit: DNR | **Implicit: Resuscitate** |
| Unknown | 6 $a_1$ | 8 $a_4$ | 38 $b_1$ $b_4$ |
| Severely Limited | 13 $a_2$ | 29 $a_5$ | 55 $b_2$ $b_5$ |
| Somewhat limited | 20 $a_3$ | 33 $a_6$ | 160 $b_3$ $b_6$ |
| **Totally independent** | 51 $c_{1-3}$ | 42 $c_{4-6}$ | 518 $d_{1-6}$ |

Analysis of this table resulted in 6 odds ratios as reported in Table 6. For ease of discussion, we will number these odds ratios as follows: $OR_1 = 1.60$, $OR_2 = 2.40$, $OR_3 = 1.27$, $OR_4 = 2.60$, $OR_5 = 6.50$, and $OR_6 = 2.54$. We can now compute these same odds ratios by substituting the appropriate frequencies into a modified version of the formula shown above:

$$OR_1 = \frac{a_1 \div b_1}{c_1 \div d_1} = \frac{6 \div 38}{51 \div 518} = 1.60 \qquad OR_4 = \frac{a_4 \div b_4}{c_4 \div d_4} = \frac{8 \div 38}{42 \div 518} = 2.60$$

$$OR_2 = \frac{a_2 \div b_2}{c_2 \div d_2} = \frac{13 \div 55}{51 \div 518} = 2.40 \qquad OR_5 = \frac{a_5 \div b_5}{c_5 \div d_5} = \frac{29 \div 55}{42 \div 518} = 6.50$$

$$OR_3 = \frac{a_3 \div b_3}{c_3 \div d_3} = \frac{20 \div 160}{51 \div 518} = 1.27 \qquad OR_6 = \frac{a_6 \div b_6}{c_6 \div d_6} = \frac{33 \div 160}{42 \div 518} = 2.54$$

From these computations, it is clear that each of these odds ratios is computed relative to the final level of the predictor variable (i.e., *Totally independent* in this case), and relative to the final level of the dependent variable (*Not Established: Assumed Full Code* in this case).

When there are 2 or more predictors, the odds ratios produced by the polychotomous regression cannot be computed this way, because the regression partials out the effects of the other variables in the model. For the odds ratios in Table 8, for example, the odds ratios for employment status are corrected for age (i.e., the effect of age is partialled out); and the odds ratios for age are corrected for employment status (i.e., the effect of employment status is partialled out).

<u>Computation and interpretation of odds ratios in polychotomous logistic regression: Short version</u>

*(Here's a shorter version . I prefer the first one.)*

The odds ratios produced by polychotomous logistic regression (e.g., Table 6) are computed with reference to the final code status group, "Not Established: Assumed Full Code"; and with respect to the final level of the particular predictor variable. When the predictor is prior functional status, for example, the reference category is "Totally independent", as shown below. (Reference categories are indicated with **bold** typeface.) If there is only one predictor variable, these odds ratios can be computed in a fashion that is similar to that used for 2x2 tables. In the context of a 2x2 table, from a conceptual point of view,
are identical to the odds ratios (ad/bc) for the following table, where one steps through all combinations of the Row 1 and Column 1 possibilities. (There would be 6 combinations in this case: 3 levels of functional status, and 2 levels of code status. These levels are indicated with *italics*.) To compute the OR of 1.60 shown in Table x, for example, let Row 1 in the following table be *unknown*, let Column 1 be *Established: Full Code*, fill in the appropriate frequencies, and compute OR = (ad/bc). Or to compute the OR of 6.50, let Row 1 be *severely limited*, let Column 1 by *No Code*, fill in the appropriate frequencies, and compute OR = (ad/bc).

| | Explicitly established: *Full Code/No Code* | **Not Established: Assumed Full Code** |
|---|---|---|
| Functional status = *unknown/severely limited/somewhat limited* | *a* | *b* |
| **Functional status = totally independent** | *c* | *d* |

When there are 2 or more predictors, the odds ratios produced by the polytomous regression cannot be computed this way, because the regression partials out the effects of the other variables in the model.